A Common Jump Factor Stochastic Volatility Model

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Abstract

We introduce a new multivariate stochastic volatility model, based on the presence of a common factor with random jumps. In this model the volatility stochastic process in each market is the sum of this common factor and a specific transitory factor. The common factor is parameterized as a permanent component, formulated as a compound binomial process. This formulation allows to capture sudden changes in the pattern of volatility that are not correctly identified by the usual models.

This stochastic volatility model can capture common jumps in the latent volatility between different markets, with particular relevance to capture economic changes and common crises in emerging markets. The model is applied to the Real/Dollar and Turkish Lira/Dollar exchange rates. The results indicate that the model adequately captures the common movements between these two markets, and allows to correctly identify the permanent and transient patterns in the volatility of these series.

Keywords: Stochastic volatility, MCMC, Jump process, Regime Changes.

JEL C53, E43, G17.

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1. Introduction

The modeling of volatility in financial assets is a key topic in finance. Volatility is an essential input for asset pricing, risk management and portfolio management, as volatility is considered as the most important measure of risk. Due to the importance of this measure there is an extensive literature proposing models to capture the volatility patterns in financial assets.

In particular these models attempt to capture some stylized facts present in this series, as the presence of clusters of volatility, heavy tails and extreme events and especially the pattern of temporal dependence in the conditional variance process. The two main classes of models of conditional volatility models are the ARCH and its generalizations proposed by Engle (1982) and Bollerslev (1986), and the stochastic volatility (SV) models, with the most common version proposed in Taylor (1986). In these two models is the fundamental component is the autoregressive structure observed in the volatility process.

In the class of ARCH models the conditional variance is a deterministic function of the past squared errors and the lagged conditional variance, while in stochastic volatility models the log-volatility is modeled as an autoregressive process. The main difference between the two classes of models is the stochastic component that is included in the SV models and absent in ARCH models.

This difference is crucial, since the SV models capture the fact that the future volatility is not a deterministic process, but subject to unpredictable shocks. By assuming that future volatility is a deterministic function of past information, the ARCH models allow building simple procedures to evaluate the likelihood function of the process, since in this way the process does not depend on unobserved factors. In stochastic volatility models the presence of the stochastic component complicates the evaluation of the likelihood function, since that conditional on information in period $t$ the volatility process is a latent factor.

The evaluation of the likelihood function in this case involves the marginalization of all unobserved stochastic volatilities, which corresponds to the need of to compute a multiple integral of dimension equal to the sample size, a problem that usually requires the use of numerical methods based on simulation procedures, as importance sampling (Monte Carlo maximum likelihood) or Bayesian estimation using methods based on Markov Chain Monte Carlo (MCMC).

Although the SV class of models is computationally more complex at first, they exhibit some advantages related to the ARCH family. Their analytical properties are simpler due to autoregressive structure of the volatility process. So it is easier to calculate measures of temporal dependence, and these models also have properties of temporal aggregation. Asymptotic properties of general ARCH models are usually more complex, and these models not have properties of temporal aggregation. Another important advantage
of stochastic volatility models is that they exist trivially in continuous time, which does not occur with ARCH models. Thus stochastic volatility models are the natural choice in asset pricing models formulated in continuous time.

It is also important to note that due to the large computational power available today stochastic volatility models can be estimated in a timely manner using a wide class of methods. Several programs have already implemented algorithms for the estimation of these models, using methods of quasi-maximum likelihood, importance sampling, and several MCMC methods. Thus, there is no computational limitation to its use. Discussions of the theoretical properties, applications and computational implementation of this class of models can be found in Ghysels et al. (1996), Broto and Ruiz (2004), Shepard and Andersen (2009) and Jungbacker and Koopman (2009). The most common form of discrete time stochastic volatility model is the log-normal formulation, which can be specified as:

\[ y_t = \exp \left( \frac{h_t}{2} \right) \varepsilon_t \]

\[ h_{t+1} = \alpha + \phi h_t + \sigma_v \upsilon_t, \]

where \( y_t \) is the observed series, usually mean adjusted returns of a financial asset, \( h_t \) is the latent volatility process, \( \varepsilon_t \) and \( \upsilon_t \) are independent standard Gaussian innovation processes, \( \alpha \), and \( \phi \), and \( \sigma_v \) are parameters. As put earlier, the main point of the SV model is the formulation of latent process \( h_t \) as a first-order autoregressive process.

While SV models have generally a good fit to the data, assuming that the innovation process \( \upsilon_t \) is from a continuous variable Gaussian the probability of large variations in the volatility process are small, since they would be associated with the probability of extreme events in that Gaussian distribution. So these models can not adequately capture jumps in the volatility, i.e., discontinuous changes in the process. In this respect the standard stochastic volatility models may be inadequate to capture the behaviors seen in emerging markets, characterized by the presence of crises, contagion effects and exposure to external shocks. In these markets is common to observe sudden changes in the pattern of volatility and the transmission of shocks and crises between markets, as discussed in Allen and Gale (2000), Anderson et al. (2010), Billio and Caporin (2005) and especially for emerging countries in Arruda and Valls Pereira (2013).

Another important problem in stochastic volatility models is the possibility of changing parameters in the volatility process. As discussed above, a persistent change in the level of volatility is not consistent with
the standard SV model that assumes constant parameters in time. In this way this model shocks has no permanent effects, and even so shocks associated with seizures or permanent changes in the economy may not affect permanently the process of volatility, which is assumed as mean reverting. An important problem in this situation is that the application of a standard SV model in the presence of structural changes can lead to incorrect results, especially spurious persistence patterns, as discussed in Hwang et al. (2007) and Messow and Krämer (2013). In this situation the structural breaks induce a positive bias in the persistence parameter $\phi$.

To circumvent this problem were formulated some alternative specifications of the SV model. In Hwang et al. (2007) is introduced a version of the SV model allowing for the possibility of regime changes controlled by a Markov chain (Markov Switching Model). This formulation allows to specify different sets of parameters in the SV model, where the chosen vector in a certain moment of time is determined by an unobserved variable determined by a discrete Markov chain, with a fixed number of regimes. As the authors discuss, this parameterization solves the problems of spurious persistence in the presence of structural changes.

However, this specification follows the usual limitation of Markov Switching models, which is the specification of a fixed number of regimes and the assumption that the Markov chain is recurrent, so is assumed a positive probability of returning to a previous regime. Another limitation in the application of this model is that it is formulated only in a univariate specification, and thus it is not possible to identify common patterns in several markets using this model.

An alternative formulation of the SV model with the possibility of level changes has been proposed in Qu and Perron (2013). In this model the stochastic volatility process is decomposed into two factors, one factor being a first autoregressive process, and the second factor an permanent process for the level formulated as a compound binomial process. Specifically, Qu and Perron (2013) present the model as follows:

$$y_t = exp \left( \frac{h_t}{2} + \frac{\mu_t}{2} \right) \varepsilon_t$$  \hspace{1cm} (3)

$$\mu_{t+1} = \mu_t + \delta_t \sigma \eta_t$$ \hspace{1cm} (4)

$$h_{t+1} = \phi h_t + \sigma v_t$$ \hspace{1cm} (5)

In this parameterization process $h_t$ has the usual interpretation of an autoregressive process, in this case capturing the transitory component of the volatility. A fundamental contribution of this model is to introduce
an additional component to model the volatility level as an additional latent factor $\mu_t$ through a compound Binomial process. In this specification the variable $\delta_t$ represents the realizations of a Bernoulli process with success probability $p$ representing the jumps in the level of volatility. In this specification if this Bernoulli process assumes zero value in the period $t$ the level the volatility $\mu_t$ is equal to the value of period $\mu_{t-1}$. If the variable $\delta_t$ takes the value one, associated with a jump in volatility, the value of $\mu_t$ is given by the value at $\mu_{t-1}$ plus a shock given by a Gaussian component with volatility $\sigma_\delta$, capturing the random intensity of jumps in volatility. Qu and Perron (2013) apply this model for the returns of the S&P 500 and NASDAQ indexes, and show that this process can adequately capture the jumps and level changes observed in these two markets.

Note that the model proposed in Qu and Perron (2013) can also be interpreted as regime switching model where there are not a fixed number of regimes and so each level change associated with a jump can be considered new regime. This parameterization removes some of the usual limitations of Markov Switching models, like the need to specify a priori the number of regimes and the fact that regimes should be recurrent. However, the model proposed by Qu and Perron (2013), in the same way of Hwang et al. (2007) model, is specified only in a univariate formulation.

In this work we propose a multivariate version of the Qu and Perron (2013) model, where we assume a special interpretation for the component $\mu_t$ as a common factor in the volatilities of different markets. This specification is particularly interesting because it allows to associate existing common patterns in the volatility of returns from various markets to this common factor, which captures the permanent component. In this parameterization the volatility of each market is obtained as the sum of this common factor plus a transitory component specific to each market. This model is especially interesting to capture the patterns of volatility in emerging markets, since the jumps this common factor may be associated with transmission of large shocks, crises and contagion between these markets.

To show the application of this new stochastic volatility model we perform the joint modeling of stochastic volatilities of daily exchange rate series of Brazilian Real/Dollar and Turkish Lira/Dollar in the period 2006-2014. The results indicate that this model can capture the common dynamics in the volatility of these two series, and also get an equivalent or higher fit to those obtained by standard SV and univariate jump models.

This work is structured as follows: in Section 2 we introduce the model of common jump factor stochastic volatility and discuss the methodology of Bayesian estimation using MCMC proposed. In 3 section we show the results obtained with the estimation of the model for the series of exchange rates. Some comparisons between the results of the proposed multivariate model with the standard SV and univariate versions of the
jump model and showed in Section 4. The final conclusions are in Section 5.

2. Common Jump Factor SV Model

The model proposed in this work is based on a multivariate extension of the random jumps in the level proposed in Qu and Perron (2013). The proposed parameterization is obtained by introducing a specific transitory factors for each observed series in the model, and assuming that each series is a combination of this autoregressive factor with a common factor for all series, using the compound Binomial process for the dynamics of this common factor. Thus we can specify the proposed model, at a bivariate specification, by the following equations:

\[
y_{1t} = \exp\left(\frac{h_{1t}}{2} + \frac{s_1 \mu_t}{2}\right) \varepsilon_{1t} \tag{6}
\]

\[
y_{2t} = \exp\left(\frac{h_{2t}}{2} + \frac{s_2 \mu_t}{2}\right) \varepsilon_{2t} \tag{7}
\]

\[
\mu_{t+1} = \mu_t + \delta_t \sigma_\eta \eta_t \tag{8}
\]

\[
h_{1t+1} = \phi_1 h_{1t} + \sigma_{v_1} v_{1t} \tag{9}
\]

\[
h_{2t+1} = \phi_2 h_{2t} + \sigma_{v_2} v_{2t} \tag{10}
\]

In this bivariate system \(y_{1t}\) and \(y_{2t}\) denote the observed series, usually mean adjusted returns of financial assets. The specific processes that capture the transitory component in the volatility processes are given by \(h_{1t}\) and \(h_{2t}\), parameterized as first-order autoregressive processes, with persistence parameters \(\phi_1\) and \(\phi_2\) and volatility \(\sigma_{v_1}\) and \(\sigma_{v_2}\). The compound Binomial process is given by the process \(\mu_t\), and the process \(\delta_t\) is a sequence of independent Bernoulli processes with common parameter \(p\), representing the probability of jumps.

In this representation the process \(\mu_{t+1}\) depend on the realization of \(\delta_t\) variable. If this variable takes value zero, the variable \(\mu_{t+1}\) remains the same as the prior period (\(\mu_t\)). If the realization of the variable \(\delta_t\) is one, featuring a jump, the process \(\mu_{t+1}\) is given by the value \(\mu_t\) plus a innovation from a Gaussian distribution, with volatility given by the parameter \(\sigma_\eta\), which represents the random size of the jumps.
We assume that the innovation processes $\varepsilon_{1t}$, $\varepsilon_{2t}$, $\eta_t$, $\nu_{1t}$ and $\nu_{1t}$ are independent Gaussian random variables. The conditional volatility for each series is obtained by adding the common jump factor multiplied by a scaling parameter $s_t$, plus each specific autoregressive component.

In this process, the conditional volatility of returns is obtained as the sum of these two components, enabling a very intuitive interpretation of the volatility process. Jumps can be associated with unforeseen events that change permanently the level of the volatility process, and thus may be associated with negative news or crises, while the autoregressive process captures the mean reverting behavior of non-permanent shocks. Thus we can also understand similarly to the models of Qu and Perron (2013) as a regime change model where the number of regimes and the volatility in each regime are data driven, and so this specification gives greater flexibility to the volatility process.

To perform the inference procedure of inference we use Bayesian estimation using Markov Chain Monte Carlo methods, similar to that described in Qu and Perron (2013). This procedure is based on the usual MCMC methods used in Bayesian estimation of stochastic volatility models, with the main change being the use of a data augmentation method for sampling the jump process, introducing a new latent variable to capture this process. In this way the Bernoulli variable takes the value one when this latent variable is greater than a certain threshold, calibrated so that the probability of exceeding this threshold is equal to the probability of the jumps in the volatility process. We also modify the MCMC procedure to directly sample the model using directly equations (6)-(10), without the linearization used in Qu and Perron (2013), by using a Metropolis-Hastings step in this stage due to the non-linearity. This avoids the use of the mixed normal approximation to the Chi-Squared distribution used by Qu and Perron (2013). Other details of the MCMC procedure are standard, and can be seen in Jacquier et al. (1994) and Qu and Perron (2013).

We use a prior scheme similar to the used in Qu and Perron (2013). We assume that the data augmentation process determining the Bernoulli thresholding is given by a sequence of independent standard Uniform densities; we same Beta prior for the probability of jumps in the model, and a Gamma process for the volatility of jumps, independent Gaussian distributions for the persistence parameters $\phi_i$ and Gamma densities for all the volatility parameters. The inference procedure is based on a burn-in of 8000 samples, and calculating the posterior distributions for latent factors and parameters using 12000 additional samples$^1$.

$^1$Implementation details, the complete structure of priors, fit and convergence measures are not presented for space reasons but can be obtained from the authors.
3. Application to Exchange Rates

To show an application of the proposed model, we use the return series of Real/Dollar and Turkish Lira/Dollar exchange rates for the period between 04/04/2005 to 27/02/2014, with a sample size of 2188 observations. These two series are chosen because of their common characteristics as exchange rates of emerging markets. Figure 1 shows the temporal evolution of these two exchange rates. It is quite evident that there are common patterns in the volatility of these two series. We observe rapid increases and similar clusters of volatility, especially in 2006 and 2008. As we mentioned this characteristic can be explained by the exposure of these two economies to common shocks in the global economy and their respective contagion/transmission.

Figura 1: Exchange Rates - Real/Dollar and Turkish Lira/Dollar

In Figure 2 we show histograms and a Gaussian approximation for the densities of the BRL/USD and TRY/USD series, and Table 1 show the respective descriptive statistics. We can see that the two series reproduce the stylized facts of financial time series in the presence of stochastic volatility, with heavy tails and a noticeably non-Gaussian distributions, as is evident by the results of Jarque-Bera test (JB) who rejects
the null hypothesis of normality for the two processes.

Figura 2: Exchange Rates - Real/Dollar and Turkish Lira/Dollar - Densities

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Max</th>
<th>Min</th>
<th>Sd.</th>
<th>Skew.</th>
<th>Kurt.</th>
<th>JB</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real/Dollar</td>
<td>4.19e-06</td>
<td>-0.00032</td>
<td>0.00891</td>
<td>-0.09214</td>
<td>0.00958</td>
<td>0.52524</td>
<td>16.1212</td>
<td>15796.40</td>
<td>0.00000</td>
</tr>
<tr>
<td>Turkish Lira/Dollar</td>
<td>0.00021</td>
<td>0.00000</td>
<td>0.06937</td>
<td>-0.06269</td>
<td>0.00874</td>
<td>0.58507</td>
<td>11.2618</td>
<td>6214.27</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

Tabella 1: Descriptive Statistics

The main results of the estimation of the common jumps factor SV model are shown in the following figures. In all figures with the latent factors we present the posterior mean and the credible interval of 95%, calculated from the simulated chains by the Bayesian estimation procedure using MCMC. In Figure 3 we present the common factor $\mu_t$ estimated for the two series, and in Figure 4 the smoothed probability of jumps, corresponding to the process compound Binomial process $\delta_t$.

We can notice from these two figures that common jumps in the volatilities of Real and Lira occur in
the years 2005, 2006, 2008 and 2013. To facilitate the visualization of these jump we showed in Table 2 all jumps with posterior probability larger than .5, corresponding to the changes in the permanent component $\mu_t$. These jumps can be associated with common shocks observed on these dates.

<table>
<thead>
<tr>
<th>Date</th>
<th>Prob. Jump</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005-05-27</td>
<td>0.6905</td>
</tr>
<tr>
<td>2006-05-11</td>
<td>1</td>
</tr>
<tr>
<td>2006-05-30</td>
<td>0.51</td>
</tr>
<tr>
<td>2006-09-06</td>
<td>0.6795</td>
</tr>
<tr>
<td>2006-09-27</td>
<td>1</td>
</tr>
<tr>
<td>2008-05-09</td>
<td>1</td>
</tr>
<tr>
<td>2013-08-14</td>
<td>0.548</td>
</tr>
</tbody>
</table>

Tabela 2: Major Jumps

In particular, we can interpret the jumps occurred in 2006 with the period of turmoil in the stock markets. According to the IMF Global Financial Stability Report this period of turbulence has origins in the concerns and expectations about the inflationary pressures from rising prices in commodities and oil, followed by a sharp reduction in the level of growth in developed economies. Expectations about the level of monetary tightening needed to control inflation led to increased levels of risk aversion of the agents, which led to rapid falls in asset prices, increases in future interest rates and a remarkable increase in the volatility of financial assets. This caused a process of contagion to emerging economies, in particular by increasing the volatility of exchange rates in these countries, as observed in the exchange series studied in this article.

The jump occurred in 2008 can be associated with the initial transmission of the crisis in the mortgage sector in the US, especially the collapse of Bear Stearns and the initial events of the global financial crisis.
of 2008-2009. However, this jump has a reduced effect in the magnitude of permanent component, and the effects of the financial crisis in the years 2008-2009 was captured by the transitory component of volatility process according to the results obtained by the common jump factor model. This transient nature may be interpreted by the fact that the strength of emerging countries in this period were later important in reducing the effects of the global financial crisis, which supports the interpretation that the global crisis did not have permanent effects on the volatility of exchange rates analyzed.

However we can identify a significant change in the common component with the jump observed in 2013. This jump can be associated with the FED signalization about on the reduction and termination of financial stimulus and the politic of zero bound in U.S. interest rates. Expectations about the end of the U.S. stimulus policy was transmitted to emerging markets, causing large devaluations in the exchange rates of emerging countries, especially for the Turkish Lira and Brazilian Real in this period. These devaluations were accompanied by high volatility in these markets. Another factor that directly impacts the economies of emerging countries in this moment are the growing concerns about the growth rate of the world economy, in special the reduction on growth rate of China. This raised expectations about the future demand for commodities, with direct impacts on foreign exchange markets.

Note that this increase in volatility was tackled in Brazil through direct interventions in the spot and futures markets rates, such as cambial swaps policy implemented in Brazil to explicitly reduce exchange rate volatility, which helps explain the observed reduction in the permanent component of volatility in late 2013 after the initial shocks.

In Figures 5 and 6 we present the transitory components $h_{1t}$ and $h_{2t}$ for Brazil and Turkey. Some
important patterns that can be observed in these series are the reduction in the transitory components observed after the turbulence of 2006, which proved short-lived, and as mentioned above fact that the nature of the crisis of 2008 was largely short lived in these exchange markets and does not represent a significant change in the permanent level component.

Figura 5: Transitory Component - Real/Dollar

![Figure 5: Transitory Component - Real/Dollar](image)

Figura 6: Transitory Component - Turkish Lira/Dollar

![Figure 6: Transitory Component - Turkish Lira/Dollar](image)

The estimates of the overall volatility process using the common jumps factor model is shown in Figs 7 and 8, comparing the estimated volatility with the absolute returns of each series, a usual proxy for the true latent volatility. We can observe that the model of common jumps can properly recover the patterns of variation observed throughout this period, especially the moments of high variability in returns observed in
crisis periods, especially in 2006, 2008 and 2013. We present several statistical measures of model fit in Table 3 in the next section, showing the performance of the common jump factor model over the standard SV and the univariate jump models.

The posterior distribution of the estimated parameters of the common jump process is placed in Figures 9 and 10, showing the distributions for the probability of jumps $p$ in the compound Binomial process and the parameter $\sigma_\eta$, which measures the intensity of the jumps in the process $\mu_t$. We note that the probability of jumps is relatively small, indicating that in the observed sample would be expected about 8 jumps, which would indicate a jump in each 274 days in average. This result is consistent with the 7 jumps identified in the series $\delta_t$, shown in Table 2. The measure of volatility jumps is also consistent with the observed jumps in the series $\mu_t$.

The posterior distribution of the persistence parameters $\phi_1$ and $\phi_2$ is shown in Figures 11 and 12. As usual in stochastic volatility models the persistence of shocks is quite high, with their posterior means being close but less than one, consistent with the mean reversion property assumed for this process. So even though these shocks eventually dissipate, the half-life of shocks in this transitory process is quite high, which is consistent with the observed volatility in assets related to emerging markets.

However, the estimation of the common jump process alters the properties of the persistence parameter. As will be discussed in section 4, the posterior distribution of the persistence parameters in the model with jumps in the common factor is on average smaller and have lower variability compared to estimates of these parameters in the model with no common factor.

Finally the scaling parameters, which determine the loading in the permanent process $\mu_t$ in each series
Figura 8: Fitted Volatility x Absolute Returns - Turkish Lira/Dollar

Figura 9: Posterior Probability - Jump Probability in Compound Binomial Process
Figura 10: Posterior Probability - Jump Volatility

Figura 11: Posterior Probability - Persistence of Transitory Component - Real/Dollar
are shown in Figures 13 and 14. The average value of the scaling parameter for Brazil is close to 1, while for Turkey this parameter is close to .86. As the process $\mu_t$ is measured in log and takes negative values, a smaller parameter indicates an increased exposure (permanent effect on the final volatility component) greater, indicating that the component $\mu_t$ is more important for Turkey than Brazil.

One way to interpret this result is through a comparison between the estimated volatility and transitory component, which captures the systematic volatility not explained by the common factor $\mu_t$. This comparison can be seen in Figure 15. The transitory component is generally higher in Brazil than in Turkey in the observed sample, and even having less exposure to permanent component $\mu_t$, the volatility explained by the model to Brazil in general is higher than in Turkey in most of the sample, except in part of the period between 2006 and 2007. Thus, although the process of exchange rate volatility is higher in Brazil, this volatility is in greater part transitory, while the permanent component is more relevant to the volatility of Turkey in this period.

4. Comparative Analysis

In this section we present some comparative analyzes of the common jump factor over the standard SV model, and also with the univariate estimation of the jump model of Qu and Perron (2013). Such analysis is needed to verify some possible limitations of the common jumps model. The main issue is that the model of common jumps imposes an important restriction on the dynamics of the volatility of each market, assuming that changes in the common component are associated with common jumps between markets.
Figura 13: Scale Parameter - Real/Dollar

Figura 14: Scale Parameter - Turkish Lira/Dollar
If this restriction is incorrect and specific jumps are most relevant the volatility fitting may be impaired. Similarly, it is necessary to check whether this restriction of permanent factor is valid, since it imposes a dynamic of changes in the level of volatility of the various series. Another relevant issue is whether the estimation process with common jumps alters the structure of persistence relative to the univariate model.

To answer these questions we show some pictures and measurements comparing the fit of different specifications for the series of exchange rates of Brazil and Turkey. Figures 16 and 17 show the process of volatilities obtained by the standard SV model without jumps and the common jumps model for Brazil and Turkey. We can see that the fitted volatilities of the standard and the common jumps model is quite similar, but that the common jumps model can explain the most extreme values of the volatility process, which is an intrinsic feature of this model due to the jumps. So we can visually see that the imposition of a common process of jumps is not an invalid restriction on the adjustment of the model.
Figura 16: Fitted - Standard SV x Multivariate Model - Real/Dollar

Figura 17: Fitted - Standard SV x Multivariate Model - Turkish Lira/Dollar
Figura 18: Fitted - Univariate x Multivariate Model - Real/Dollar

Figura 19: Fitted - Univariate x Multivariate Model - Turkish Lira/Dollar
It is also relevant to compare the fit of the model with common jumps with the original univariate model of jumps proposed by Qu and Perron (2013), applied separately for each exchange rate. For this we compare the latent factors generated by univariate and multivariate specifications. Figures 18 and 19 show the fitted volatilities process by the two models. In this case we note that the adjustment to the exchange rate in Brazil in the two specifications are very similar, while there is a more obvious difference in the fit of the two models for Turkey.

For quantitative measurements of these differences, we present in Tables 3 the measures of mean error (me), root mean square error (rmse) and mean absolute error (mae) of three specifications analyzed (univariate jump model, multivariate common jumps model and standard SV model), comparing the fitted model volatilities with the usual proxy for the unobserved true volatility, the absolute returns of series.

<table>
<thead>
<tr>
<th></th>
<th>ME</th>
<th>RMSE</th>
<th>MAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BRL Univ.</td>
<td>-0.001457</td>
<td>0.005690</td>
<td>0.004076</td>
</tr>
<tr>
<td>BRL Multi.</td>
<td>-0.001053</td>
<td>0.005641</td>
<td>0.003966</td>
</tr>
<tr>
<td>BRL Stand.</td>
<td>-0.001152</td>
<td>0.005809</td>
<td>0.004088</td>
</tr>
<tr>
<td>LIRA Univ.</td>
<td>-0.001475</td>
<td>0.005380</td>
<td>0.003969</td>
</tr>
<tr>
<td>LIRA Multi.</td>
<td>0.000067</td>
<td>0.005192</td>
<td>0.003595</td>
</tr>
<tr>
<td>LIRA Stand.</td>
<td>-0.001165</td>
<td>0.005241</td>
<td>0.003853</td>
</tr>
</tbody>
</table>

Tabela 3: Fit Results for the Observed Volatility

The results in this table show that the multivariate model of common jumps represents gains in mean error (bias), rmse and mae towards the univariate jump model and the standard SV model without jumps, for the exchange rates in Brazil and Turkey, showing that the model of common jumps dominates the two alternative models in all measured aspects. This result is especially relevant comparing the common jumps with the univariate jumps model.

The latter model has more flexibility, since the model estimated a distinct jump process in level for each market, and so could get a better fit given the largest number of parameters. However, the results in Table 3 show that not only the imposition of a dynamic of common jumps does not represent a loss of fit, as leads to gains in relation to other models studied.

In Figures 20 and 21 we compare the specific autoregressive and permanent factors between univariate and multivariate models specifications for the two series. One can see in Figure 20 that there is a significant difference between the estimated values of the common jumps factor factor and the factors estimated by univariate models. In the univariate estimation of the Real/Dollar exchange model is identified a greater number of jumps compared to to the common jumps model, but does not associate the volatile periods of 2006 and 2013 with jumps in the level of volatility. The univariate estimation also identifies a distinct standard
for Turkish Lira, in this case with a smaller number of jumps compared with the multivariate common jumps model, and also not associate the sudden volatility growths in 2006 and 2013 with jumps in the permanent component $\mu_t$.

As the univariate and multivariate models get a very similar fit for the volatility, the identification of distinct jumps leads to modifications in the transitory processes associated with the specific autoregressive components of each market, which are shown in Figure 21. One way to understand this difference lies in the persistence of the autoregressive components, that become more persistent in the univariate estimation.

As discussed in Hwang et al. (2007) and Messow and Krämer (2013) the existence of changes in the level of volatility by the existence of structural breaks or regime changes can generate spurious persistence in stochastic volatility models. To discuss this, we show in Figures 22 e 23 the posterior distributions of the parameter $\phi$ of the autoregressive process from the estimation of univariate and common jumps models.
Figura 21: Transitory Component - Univariate and Multivariate Models

![Transitory Component - Univariate and Multivariate Models](image1)

Figura 22: Persistence - Univariate x Multivariate Models - Real/Dollar

![Persistence - Univariate x Multivariate Models - Real/Dollar](image2)
We can notice that there is a noticeable difference between the pattern of persistence between the estimation in the univariate jumps model of Qu and Perron (2013) and the same parameter estimated in the proposed common jumps model. The parameters estimated in the univariate model have higher posterior mean and have a significant posterior probability in the region of non-stationarity of the autoregressive process, compared to the same parameters estimated by the common jumps model. In this specification, we have the persistence parameters are located almost entirely in the stationarity region (note that there is a distortion in the kernel estimation at the boundaries), as expected by the permanent - transitory decomposition of jumps in this common jumps specification.

In this model is essential that the autoregressive process be stationarity, to correctly differentiate the permanent jumps in the level of transitory shocks in the volatility process. In this analysis, there is evidence that the univariate jump model may be subject to spurious persistence problem discussed in Hwang2007 and Messow and Krämer (2013), even with models including a mechanism for possible changes in the level process.

5. Conclusions

In this work we introduce a new multivariate stochastic volatility model, based on the presence of a common factor that captures mutual patterns in the level of latent volatilities. This factor is modeled as a compound Binomial process that identifies possible jumps and discontinuities in the level. This model allows to correct some existing limitations in the standard SV models, in particular the failure to capture sudden and persistent changes in the latent volatility process.
This model incorporates elements of several classes of models used in finance, such as the use of jumps processes, typically used in asset pricing (jump diffusion models), but interpreted as a common factor among all series analyzed. This specification can also be interpreted as a model of regime changes, but where the number of regimes is not fixed a priori and determined through the common series analyzed. The common jumps model also allows to avoid the problem of spurious persistence in volatility caused by changes of parameters in stochastic volatility models, identified in Hwang et al. (2007) and Messow and Krämer (2013).

The results obtained in the joint modeling of Real/Dollar and Turkish Lira/Dollar exchange rates show that this model can capture the main features in these two series, especially the common movements in the volatility process. The model can also identify moments of permanent changes in the level of volatility by associating jumps with times of financial contagion arising from moments of nervousness and crises in the core financial markets.

We analyze the connection of estimated jumps with the relevant economic events, and show that this model can capture important qualitative aspects of the volatility process, while this model also achieves a better fit to the dynamics of volatility. Other possible applications of the proposed model are the construction of forecasts for the volatility and the use in risk management.

References


