Watching the News: Optimal Stopping Time and Scheduled Announcements*

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Abstract

The present work studies optimal stopping time problems in the presence of a jump at a fixed time. It characterizes situations in which it is not optimal to stop just before the jump. The results may be applied to the most diverse situations in economics but the focus of the present work is on finance. In this context, a jump in prices at a fixed date is consistent with the effects of scheduled announcements. We apply the general result to the problem of optimal exercise for American Options and to the optimal time to sell an asset (such as a house or a stock) in the presence of fixed cost. In the first application we obtain that it is not optimal to exercise the American Option with convex payoff just before the scheduled announcement. For the second application we obtain that it is not optimal to sell an asset just before the announcement depending upon the utility function and/or the way the prices jump. We provide also a numerical solution for the second application in a particular case.

Keywords: Optimal Stopping Time, Scheduled Announcements, Quasi-Variational Inequality, Jump-Diffusion Models, Numerical Methods in Economics.

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1 Introduction

Several announcements are scheduled events at which the government, institutions or firms often disclose surprising news. For example, the dates of the Federal Open Market Committee (FOMC) meetings are known in advance[1] and changes in monetary policy are now announced immediately after it. The Federal Reserve Bank determines interest rate policy at FOMC meetings and according to Bloomberg website[2] " ... [the FOMC meetings] are the single most influential event for the markets.". Other macroeconomic data have their release known in advance as well, such as the GDP, CPI, PPI and others. Such information is incorporated into securities' prices very quickly. Most of the price change can be seen within 5 minutes after the announcement[3]. There are similar findings for firms as well. For example, it is common practice among listed firms to release in advance the dates of the earning announcements. Several authors find a quick move in the markets after the information is released with the bulk of price change in the first few minutes (Pattel and Wolfson (1984)).

In situations where action entails a fixed cost, the economic agents may prefer do nothing most of the time and take some action only occasionally. Empirical studies find such behavior in most diverse fields of economics[4]. Those situations are usually modeled using stochastic control with fixed cost in continuous time. Those problems are called impulse control when the agent takes several actions choosing the time of each one. When the action is taken just once, it is called optimal stopping time problem. The later problem naturally arises when pricing American Options. Oksendal and Sulem (2007) and Stockey (2009) provide a mathematical theory on those problems presenting some important models from the literature.

Our interest is to analyze optimal stopping time problems in the presence of scheduled announcements. We characterize situations where an agent prefers to wait for the information before taking an action. These results may be applied to the most diverse economic situations as the above paragraph suggests, but our focus here is on financial markets. In particular we show that it is never optimal to exercise a class of American Derivatives just before this type of announcement. This class includes very common derivatives such as American calls and puts. Moreover we study the optimal time to sell an asset (such as a house) in the presence of fixed costs and scheduled announcement. We show that it is not optimal to sell just before the announcements for some cases of utility function and/or jumps characteristics. We also provide a numerical solution for the second application in a particular case.

Several papers model security's prices as a jump-diffusion process in continuous time. The fast price change with news suggests that jumps may be used as a way to incorporate announcements in the price process. It is common to consider the jumps' time as random and unknown before it

[1] Those dates can be seen at: http://www.federalreserve.gov/monetarypolicy/fomccalendars.htm.

[2] It is written in the link at 03/19/2013: http://bloomberg.econoday.com/byshoweventfull.asp?fid=455468&cust=bloomberg-us&year=2013&lid=0&prev=/byweek.asp#top.

[3] See, for instance, Ederington and Lee (1993), Andersen and Bollerslev (1998) or Andersen et. al (2007).

[4] For instance, Bils and Klenow (2004) and Klenow and Kryvtsov (2008) documents the infrequent price changes in retail establishments and Vissing-Jorgensen (2002) finds that households rebalance their portfolio infrequently.

occur. Nonetheless scheduled announcements don't happen at random dates and they are known in advance. Then we model it as jumps occurring at a fixed and known time[5]. Other empirical findings on prices' behavior may be incorporated in similar fashion. For example, the price volatility may be modeled as an extra continuous time process jumping with news.

Note that the jump is the consequence of some information release impacting the environment or the agent's beliefs about it. In this respect, waiting for the jump is a way to gather more information before taking some action. In some cases there is no substantial risk in waiting for the information so the agent may prefer to act later. In others, waiting is risky as the information may destroy some opportunities. Such interpretation is particularly consistent with evidence in financial markets as announcements usually increase trading activity[6].

Some authors[7] study trading volume behavior around announcements considering investor with exogenous reason for selling an asset. Those investors may have time discretion and may want to avoid trade before an announcement fearing an adverse transaction with a better informed agent. We may add to this literature highlighting that such behavior may be found even without the information asymmetry. As an example, we provide the numerical result for the case in which the price follows a geometric Brownian motion, there is a fixed transaction cost, and the agent is risk-neutral and wants to sell an asset for exogenous reason.

The rest of the article is organized as follows. Section 2 presents the results for optional exercise of American Option in the presence of scheduled announcements. The characteristics of the risk neutral measure allow an easy way to prove the result and provide the basics steps for the more general propositions. Section 3 provides the main results in its generality. Section 4 provides one application with a numerical result: the optimal time to sell an asset. Section 5 presents a discussion and Section 6 summarizes the findings and points towards future work. The most technical proofs are in the appendix A and the numerical algorithm‘s details is in Appendix B.

2 Optimal Exercise for American Options

The goal of the present section is twofold: to provide a simple demonstration in a particular case and to give a contribution to the optimal exercise of American Options. We show that it is never optimal to exercise just before a scheduled announcement in some common situations. What simplifies the proof is the existence of the risk-neutral measure. The demonstration here gives the guidelines for the general case. We have one empirical implication in this section: if the agents are rational then no exercise is made a little before the announcement for American Option with convex payoff (and absence of arbitrage).

[5] Other authors have a similar modeling strategy. For instance, Dubinsky and Johannes (2006) build an option pricing model incorporating scheduled announcements as jumps occurring at a known date.

[6] There are hundreds of papers about it. It has attracted interest of diverse areas such as economics, finance and accounting. See the seminal work of Beaver (1968) and a review by Bamber et al. (2011). Recent empirical findings in finance includes Chae (2005), Hong and Stein (2007) and Saffi (2009). Some important theoretical work are: Admati and Pfleiderer (1988), Foster and Viswanathan (1990), George et al. (1994).

[7] For instance, see Admati and Pfleiderer (1988), Foster and Viswanathan (1990) or George et al. (1994).

In general, for put options there is a region in which it is better to exercise and the premium is the same as the payoff. Do not exercise at time t means a premium greater than the payoff at t. A jump in a fixed date increases the uncertainty around it and it seems reasonable that the issuer raises the premium. This would imply a smaller region of prices where it is optimal to exercise. In this sense, our results would be intuitive and its interest lays in that the exercise regions shrink to an empty set. Nonetheless, to the best of our knowledge, this reasoning is not necessarily true. For instance, Ekstrom (2004) shows that for a class of American Options the premium increases with volatility but the proposition isn't applied to American puts.

It is not straightforward to infer what happens in the neighborhood of an announcement for the exercise of American Options. Pattel and Wolfson (1979), (1981) find empirically that the implied volatility increases close to announcements, i.e., other things constant, there is an increase in the premium for Europeans calls and puts. On the other hand American calls have usually the same premium as its European counterparty. It is not the case when there are dividends payments because it may be advantageous to exercise just before the payment.

The modeling of a scheduled earning announcement as a jump is taken by Dubinsky and Johannes (2006). They consider a jump-diffusion model with stochastic volatility, apply it to a set of equities and try to measure empirically some definitions of uncertainty about the news. Similarly Pattel and Wolfson (1979), (1981) try to gauge the uncertainty with a generalization of the Black-Scholes-Merton model in which the stock volatility varies deterministically over time. In their generalization the implied volatility increases as the option approaches the announcement date, and drops to a constant after it .

2.1 Example: American Put Option on a Black-Scholes-Merton Model with Scheduled Announcement

This subsection introduces the notation and presents a concrete example. Suppose we have an American put on a equity with 60 days maturity of and that the next FOMC meeting will happen in 30 days whose decision has a far reaching impact in the industry of this equity and will define a new interest rate. The actual interest rate is 1% and suppose the uncertainty about the meeting implies an interest rate of 0.75%, 1.00% or 1.25% after it. Let T_M be the time of maturity (60 days) T_A the time of the scheduled announcement (the end of the FOMC meeting in 30 days) and S_t be the price of my equity at time t. We model the price process as in the Black-Scholes-Merton environment but with a jump in price at T_A and a change in the interest rate at T_A, i.e., the price follows geometric Brownian motion and (in the risk-neutral measure) it reads:

$$dS_t = r_t S_t dt + \sigma S_t d\widetilde{B}_t + \Delta S_{T_A} \chi_{\{t=T_A\}}, \quad (1)$$

$$S_0 = z_0 \quad (2)$$

where z_0 is a constant, $\chi_{\{t=T_A\}}$ is the indicator function

$$\begin{aligned} \chi_{\{t=T_A\}}(t) &= 0 \text{ if } t \neq T_A \\ &= 1 \text{ if } t = T_A, \end{aligned} \quad (3)$$

$$r_t = r_{BA} \text{ if } t < T_A \text{ (Before Announcement)}, \tag{4}$$

$$r_t = r_{AA} \text{ if } t \geq T_A \quad \text{(After Announcement)}, \tag{5}$$

r_{BA} is a constant, r_{AA} is a random variable whose realization is not known before T_A, σ is the constant volatility and $\widetilde{B}_t$ is the Wiener process in the risk-neutral measure. r_{AA} has a discrete distribution with 3 possible outcomes: 0.75%, 1.00% or 1.25%. Moreover, the price process is continuous before and after T_A but has a jump at T_A of

$$\Delta S_{T_A} = \zeta S\left(T_A-\right) \tag{6}$$

where $S_{(T_A)^-}$ is the left limit of the price process

$$S_{(T_A)^-} = \lim_{t \to (T_A)^-} S_t, \tag{7}$$

ζ has a lognormal distribution[8] and ΔS_{T_A} is the jump's size:

$$\Delta S_{T_A} = S_{T_A} - \lim_{t \to (T_A)^-} S_t. \tag{8}$$

In order to compute the American put's premium we shall consider the early exercise feature and that the option holder uses it optimally. As we are in the risk-neutral measure, we compute the present value expectation using the discounting

$$e^{-\int_0^\tau r_s ds} \tag{9}$$

where τ is the exercise time. If $\tau \leq T_A$ we have the discount as $e^{-r_{BA}\tau}$, otherwise we have $e^{-r_{BA}T_A - r_{AA}(\tau - T_A)}$ and the premium for a given strategy τ is

$$\widetilde{E}\left[e^{-\int_0^\tau r_s ds}\left(K - S(\tau)\right)^+\right] \tag{10}$$

where K is the strike, $\widetilde{E}\left[\cdot\right]$ denotes the expectation in the risk-neutral measure and $(x)^+ = \max\left\{0, x\right\}$. As we seek the maximum, we have

$$v(z_0) = \max_{\tau \leq T_M} \widetilde{E}\left[e^{-\int_0^\tau r_s ds}\left(K - S(\tau)\right)^+\right] \tag{11}$$

where τ is a stopping time, T_M is the maturity and $v(z_0)$ is the premium at $t = 0$ when $S(0) = z_0$.

We lost the Markov property held by the Black-Scholes-Merton model when we introduced a scheduled announcement. Nonetheless, we still have something similar. For $t < T_A$, all we know

[8] To be precise about the information structure, we shall define the probability space $\left(\Sigma, \Omega, \widetilde{P}\right)$ along with the filtration $(\mathcal{F}_t)_0^{T_M}$. Let the price process be right-continuous and the portfolios be left-continuous. The realization of ζ and r_{AA} aren't known before T_A, i.e., these information belong to $\mathcal{F}_{T_A}$ but not to $\mathcal{F}_t$ if $t < T_{A.}$

Note that we are considering only the risk neutral measure $\widetilde{P}$, i.e., we only need to know the jump size and change distributions in this measure.

about the distribution after t is contained in the price level. For conditional expectation this implies that

$$\widetilde{E}\left[\cdot|\mathcal{F}_t\right] = \widetilde{E}\left[\cdot|S_t = z\right] \quad \text{for } t < T_A. \tag{12}$$

By the other hand, after $t \geq T_A$ all information is contained in $S_t = z$ and $r_{AA} = r$ and we have

$$\widetilde{E}\left[\cdot|\mathcal{F}_t\right] = \widetilde{E}\left[\cdot|S_t = z, r_{AA} = r\right] \quad \text{for } t \geq T_A. \tag{13}$$

To what follows, we need to define the premium for other dates. For $t < T_A$ denote it by[9] $V_{BA}(t,z)$:

$$V_{BA}(t,z) = \max_{t \leq \tau \leq T_M} \widetilde{E}\left[e^{-\int_t^\tau r_s ds}\left(K - S(\tau)\right)^+ |S(t) = z\right] \tag{14}$$

and for $t \geq T_A$

$$V_{AA}(t,z,r) = \max_{t \leq \tau \leq T_M} \widetilde{E}\left[e^{-\int_t^\tau r_s ds}\left(K - S(\tau)\right)^+ |S(t) = z, r_{AA} = r\right]. \tag{15}$$

In the present work, we want to study the exercise behavior just before just before T_A and we do it through the optimal stopping time τ. A decision to stop should depends only upon the past information, i.e., if the agent wants to exercise in t this decision is make using the information $\mathcal{F}_t$. But the all relevant information is in the value of S_t (and $r_{AA} = r$ if $t \geq T_A$). Then, for each t (and r after T_A) we have a set of prices that makes optimal the exercise and in this case the premium is $V_{BA}(t,z) = (K - S(t))$. We call this the stopping set[10]

$$\mathbf{S}_{BA} = \left\{(t,z); V_{BA}(t,z) = (K-z)^+\right\} \text{ for } t < T_A, \tag{16}$$

$$\mathbf{S}_{AA} = \left\{(t,z,r); V_{AA}(t,z,r) = (K-z)^+\right\} \text{ for } t \geq T_A. \tag{17}$$

By the other side, we have the equity price region where it is not optimal to exercise, i.e., the continuation set where the premium is greater than the payoff[11]

$$\mathbf{C}_{BA} = \left\{(t,z); V_{BA}(t,z) > (K-z)^+\right\} \text{ for } t < T_A, \tag{18}$$

[9] We could do simply:

$$V(t,z,r) = \max_{t \leq \tau \leq T_A} \widetilde{E}\left[e^{-\int_t^\tau r_i ds}\left(K - S(\tau)\right)|S(t) = z, r_s = r\right]$$

considering the interest rate another process that jumps with the announcement. Nonetheless we want to emphasize the role of the announcement.

[10] Actually, the stopping set $\mathbf{S}$ shall be defined as

$$\mathbf{S} = \left(\mathbf{S}_{BA} \times r_{BA}\right) \cup \mathbf{S}_{AA}$$

where $\times$ denotes cartesian product.

[11] Again, the continuation region $\mathbf{C}$ shall be defined as

$$\mathbf{C} = \left(\mathbf{C}_{BA} \times r_{BA}\right) \cup \mathbf{C}_{AA}.$$

$$\mathbf{C}_{AA} = \left\{(t,z,r); V_{AA}(t,z,r) > (K-z)^{+}\right\} \text{ for } t \geq T_A. \tag{19}$$

In this model, it is not optimal to stop just before the announcement and we show this below. In the next subsection we give sufficient conditions for not being optimal to exercise (stop) just before the announcement for a generic model, i.e., for each z there is $\varepsilon > 0$ such that $(T_A - \varepsilon, z) \in \mathbf{C}_{BA}$.

2.2 Generic Problem

Let Z_t be a n+m-dimensional defined as:

$$Z_t = (S_t, X_t) \tag{20}$$

where S_t is a n-dimensional process for assets prices satisfying the stochastic differential equation (SDE hereafter) in the real world (objective measure):

$$dS_t = S_t \alpha(S_t, X_t, \theta_t)dt + S_t \sigma(S_t, X_t, \theta_t)dB_t + \Delta S_{T_A} \chi_{\{t=T_A\}} \tag{21}$$

X_t is a m-dimensional vector satisfying the SDE:

$$dX_t = \alpha_X(S_t, X_t, \theta_t)dt + \sigma_X(S_t, X_t, \theta_t)dB_t + \Delta X_{T_A} \chi_{\{t=T_A\}}, \tag{22}$$

B_t be a n+m-dimension Wiener process, α, α_X, σ, σ_X satisfies usual regularity conditions (see Oksendal and Sulem (2007), Theorem 1.19), $t \geq 0$ and θ_t is a set of parameters satisfying

$$\theta_t = \theta_{BA} \quad \text{Before the Announcement}, \tag{23}$$

$$\theta_t = \theta_{AA} \qquad \text{After the Announcement}, \tag{24}$$

where θ_{AA} is a random variable known after the announcement. Note that the process X_t isn't a price process. For instance, in the stochastic volatility model (as in Heston (1993) for instance) the volatility is a process but it is not a price process. It implies that it isn't (in general) a martingale under the risk-neutral measure. The process may include jumps as well but we do not consider it here explicitly in order to simplify the exposition. This broad specification includes, for instance, the Black and Scholes model, Merton model and the class of Affine Jump-Diffusion models as in Duffie et al. (2000).

The scheduled announcement is made at $T_A > 0$ and there is a jump in (S_{T_A}, X_{T_A}) :

$$\Delta S_{T_A} = S_{T_A} - \lim_{t \to (T_A)^-} S_t, \tag{25}$$

$$\Delta X_{T_A} = X_{T_A} - \lim_{t \to (T_A)^-} X_t, \tag{26}$$

along with a change in the parameters as

$$\theta_t = \theta_{BA} \text{ for } t < T_A, \tag{27}$$

$$\theta_t = \theta_{AA} \text{ for } t \geq T_A \tag{28}$$

with θ_{AA} known only for $t \geq T_A$.

We assume that there is a risk-neutral measure. In the absence of arbitrage this is indeed true (see, for instance, Duffie (2001)). Under this measure, we have that the asset prices satisfies the SDE:

$$dS_t = r_t S_t dt + S_t \sigma(S_t, X_t, \theta_i) d\widetilde{B}_t + \Delta S_{T_A} \chi_{\{t=T_A\}} \tag{29}$$

and X_t:

$$dX_t = \widetilde{\alpha}_X(S_t, X_t, \theta)dt + \sigma_X(S_t, X_t, \theta_i)d\widetilde{B}_t + \Delta X_{T_A}\chi_{\{t=T_A\}}. \tag{30}$$

where r is the instantaneous interest rate assumed constant for simplicity[12], B_t be a n+m-dimension Wiener process in the risk neutral measure and $\widetilde{\alpha}_X$, σ_X satisfies regularity conditions ((see Oksendal and Sulem (2007), Theorem 1.19)). We assume further that the jump at T_A, ΔZ_{T_A},is a random variable that depends only upon $Z\left(T_A-\right)$ (as in the multiplicative case of equation (**??**)) and that the future distribution of the economy only depends upon the actual state of the economy. We express the last assumption with the equation:

$$\widetilde{E}\left[\cdot|\mathcal{F}_t\right] = \widetilde{E}\left[\cdot|\left(S(t), X(t)\right) = z, \theta_t = \theta\right]. \tag{31}$$

where z is a n+m dimensional constant and θ is a constant set of parameters.

The price of American Option is obtained defining an optimal stopping problem in the risk neutral measure. Let $g : R^n \to R$ denote the option's payoff and let $T_M > T_A$ be the maturity. Then we have for the option's premium:

$$V_{BA}(t,z) = \max_{t \leq \tau \leq T_M} \widetilde{E}\left[e^{-\int_t^\tau r_s ds} g\left(S_\tau\right)|Z(t) = z\right] \quad \text{for } t < T_A, \tag{32}$$

$$V_{AA}(t,z,\theta) = \max_{t \leq \tau \leq T_M} \widetilde{E}\left[e^{-\int_t^\tau r_s ds} g\left(S_\tau\right)|Z(t) = z, \theta_{AA} = \theta\right] \quad \text{for } t \geq T_A, \tag{33}$$

where V is the premium. Note that we make the assumption that g only depends upon S_t.

2.3 Results for Convex American Options

The simplification in the American Option case comes mainly by two simple equalities we stablish now. The prices and the premium follow a martingale in the risk-neutral measure. In particular, for $t < T_A \leq u$ we have[13]

$$e^{-rt} y = \widetilde{E}\left[e^{-ru} S_u | Z_t = (y,x)\right] \quad \text{for } t < T_A \leq u, \tag{34}$$

[12] We can model the interest rate process as well as in done the example above. Nonetheless nothing changes in the proof of the proposition.

[13] In the general case we should use $e^{-\int_0^t r_s ds}$ instead of e^{-rt}.

$$e^{-rt}V_{BA}(t,z) = \widetilde{E}\left[e^{-ru}V_{AA}(u,Z_u,\theta_u)|Z_t = z\right] \quad \text{for } t < T_A \leq u. \tag{35}$$

If $u = T_A$ we can make the limit:

$$\begin{aligned} e^{-rt}y &= \widetilde{E}\left[e^{-ru}S_u|Z_t = (y,x)\right] \\ \lim_{t\to(T_A)^-} e^{-rt}y &= \lim_{t\to(T_A)^-} \widetilde{E}\left[e^{-ru}S_u|Z_t = (y,x)\right] \\ e^{-rT_A}y &= \widetilde{E}\left[e^{-rT_A}S_{T_A}|Z_{(T_A)^-} = (y,x)\right] \end{aligned} \tag{36}$$

or[14]

$$y = \widetilde{E}\left[S_{T_A}|Z_{(T_A)^-} = (y,x)\right] \tag{37}$$

and for the same reason

$$\lim_{t\to(T_A)^-} V_{BA}(t,z) = \widetilde{E}\left[V_{AA}(T_A, Z_{T_A}, \theta_{T_A})|Z_{(T_A)^-} = z\right]. \tag{38}$$

The above 2 equations is what make the proof easier. We will implicitly impose that $V_{BA}(t,z)$ is continuous in t close to T_A. Although we can avoid this assumption, it simplifies the proof.

Proposition 1 *Consider the model defined in the risk-neutral measure by the equations (21)-(30) along with the distribution of θ_{AA} and the jumps in T_A. Consider further an American Option with maturity $T_M > T_A$ whose g is a convex function of S_t. Moreover, assume that it is not optimal to execise at T_A with positive probability in the risk-neutral measure. Then for each z there is $\varepsilon > 0$ such that it is never optimal to exercise the option at time $t \in (T_A - \varepsilon, T_A)$ if $Z_t = z$. In other words, it is never optimal to exercise just before the announcement.*

Proof. What we want to show is that

$$\lim_{t\to(T_A)^-} V_{BA}(t,z) > g(y) \tag{39}$$

with $z = (y,x)$ because the above limit means that exists $\varepsilon > 0$ such that

$$V_{BA}(t-\varepsilon,z) > g(y) \tag{40}$$

[14] The step where the limit enters on the expectation needs to be better defined. More explictly, make

$$\begin{aligned} \widetilde{E}\left[\cdot|\lim_{t\to(T_A)^-} Z_t = (y,x)\right] &= \widetilde{E}\left[\cdot|\lim_{t\to(T_A)^-} Z_t = (S_t,X_t); S_t = y; X_t = x\right] \\ &= \widetilde{E}\left[\cdot|\lim_{t\to(T_A)^-} \mathcal{F}_t; S_t = y; X_t = x\right] \end{aligned}$$

and we shall define $\lim_{t\to(T_A)^-} \mathcal{F}_t$ as an increasing set limit

$$\lim_{t\to(T_A)^-} \mathcal{F}_t = \cup_{n=1}^{\infty}\left(\mathcal{F}_{T_A - \frac{1}{n}}\right).$$

and the strict inequality is a suficient (and a necessary) condition to not exercise,i.e, (t, z) belongs to the continuation region.

Being not optimal to execise at T_A with positive probability implies that

$$\widetilde{E}\left[V_{AA}(T_A, Z_{T_A}, \theta_{T_A})|Z_{(T_A)^-} = z\right] > \widetilde{E}\left[g(S_{T_A})|Z_{(T_A)^-} = z\right] \tag{41}$$

because we have the strict inequality $V_{AA}(T_A, Z_{T_A}, \theta_{T_A}) > g(S_{T_A})$ with positive probability and the inequality $V_{AA}(T_A, Z_{T_A}, \theta_{T_A}) \geq g(S_{T_A})$ with certainty.

Finally, in order to obtain the inequality (39), we just need to do[15]:

$$V_{T_A -} = \widetilde{E}_{(T_A)^-}\left[V_{T_A}\right] > \widetilde{E}_{(T_A)^-}\left[g(S_{T_A})\right] \geq g\left(\widetilde{E}_{(T_A)^-}\left[S_{T_A}\right]\right) = g\left(y\right). \tag{42}$$

$$V_{T_A -} > g\left(y\right)$$

where $V_{T_A -} = \lim_{t \to (T_A)^-} V_{BA}(t, z)$ and $\widetilde{E}_{(T_A)^-}\left[V_{T_A}\right] = \widetilde{E}\left[V_{AA}(T_A, Z_{T_A}, \theta_{T_A})|Z_{(T_A)^-} = z\right]$. ■

In the Black-Scholes-Merton model without dividend payment but with this kind of news, we have that the exercise feature for American call is worthless and premium is equal to the European one with the same characteristics. Moreover, for options where the exercise feature has some value, this proposition means that the premium will increase at least in some set of prices.

A crucial assumption is the possibility of no exercise after the announcement. If you know that you will exercise anyway after the news release, why bother to wait for it? Actually it is reasonable to have at least a small chance to not exercise after the announcement. For instance, one may think that the jump has a lognormal distribution. In this case any (open) interval of S has a positive probability to occur.

On the other hand, there is a greater chance to exercise after the announcement. This is a consequence of the jump and the change in the price process at the announcement. In the next sections we analyze this more deeply. For instance, the modeling approach we use for timing the selling of an asset is quite similar to the above problem.

[15] Or, in a more complete notation, we have with $z = (y, x)$:

$$\begin{aligned}
\lim_{t \to (T_A)^-} V_{BA}(t, z) &= \widetilde{E}\left[V_{AA}(T_A, Z_{T_A}, \theta_{T_A})|Z_{(T_A)^-} = z\right] \\
&> \widetilde{E}\left[g(S_{T_A})|Z_{(T_A)^-} = z\right] \\
&\geq g\left(\widetilde{E}\left[S_{T_A}|Z_{(T_A)^-} = z\right]\right) \\
&= g\left(y\right).
\end{aligned}$$

3 Optimal Strategies Close to Announcement

We established in the previous section some results for American Options when the payoff is convex and there is a risk-neutral measure. In this section we relax those assumptions characterizing general models that use optimal stopping time with a random change at a known and fixed time. We simplify some definitions here using a notation similar to Shreve (2000) in order to have a more readable text but in Appendix A we give a full account.

Let T_A be the time of announcement, $Z_t = (Y_t, X_t)$ be a n+m-dimensional[16] process where Y_t is n-dimensional that doesn't jump at T_A a.s., and X_{T_A} is a m-dimensional process that jumps with a positive probability at T_A:

$$Z_t = (Y_t, X_t), \tag{44}$$

$$dZ_t = \alpha(Z_t)dt + \sigma(Z_t)dB_t + \Delta Z_{T_A}\chi_{\{t=T_A\}}, \tag{45}$$

$$Z(0) = z_0 \tag{46}$$

$$\begin{aligned} X(T_A) &= X(T_A-) + \Delta X(T_A) \\ Y(T_A) &= Y(T_A-) \qquad \text{a.s.} \end{aligned} \tag{47}$$

where α and σ are function satisfying some regularity conditions ensuring the existence of strong solution (see Oksendal and Sulem (2007), Theorem 1.19), B_t is a n+m-dimensional Wiener Process and $\Delta X(T_A)$ has a probability distribution depending upon the information $\mathcal{F}_{T_A-}$. Assume that the process has the properties:

$$E\left[\cdot|\mathcal{F}_t\right] = E\left[\cdot|Z_t = z\right], \tag{48}$$

i.e., all the information relevant for the distributions after t is summed up in the value of state variables at t: $(t, Z_t = z)$.

Let $f : \mathbb{R}^{n+m} \to \mathbb{R}$ and $g : \mathbb{R}^{n+m} \to \mathbb{R}$ be continuous functions satisfying regularity conditions (see Oksendal and Sulem (2007), Chapter 2) and suppose $f \geq 0$ and $g \geq 0$. The optimal stopping problem at time 0 is to find the supremum:

$$v(z) = \sup_{\tau \in \Upsilon} E^y \left[\int_0^\tau f((Z(t))dt + g(Z(\tau))\chi_{\{\tau<\infty\}} \right] \tag{49}$$

where $\chi_{\{\tau<\infty\}}$ is the indicator function and at time t:

[16] All the proofs consider a probability space $(\Omega, \mathcal{F}, P)$ and the filtration $\mathcal{F}_t$ and there is no change when $Z(t)$ is a jump diffusion in $\mathbb{R}^{n+m}$ given by

$$dZ(t) = b(Z(t), \theta(t))dt + d(Z(t), \theta(t))dB(t) + \int_{\mathbb{R}^K} \gamma(Z(t^-), z, \theta(t^-))\widetilde{N}(dt, dz), \tag{43}$$

where the jump is explicitly now. We should define also the solvency region. It is an open set $S \subset \mathbb{R}^{n+m}$. In order to simplify the exposition we consider $S = \mathbb{R}^{n+m}$ (all space) and omit it in the main text.

$$V(t,z) = \sup_{\tau \geq t} E\left[\int_t^{\tau} f((Z(t))dt + g(Z(\tau))\chi_{\{\tau<\infty\}}|Z_t = z\right]. \tag{50}$$

Note that the change in parameters here are inside the process X_t implicitly. For instance, the risk-free rate of the example in section 2 may be regarded as one of the dimensions in X_t.

We make the assumption that the random variable $\Delta X(T_A)$ depends only upon $Z(T_A-)$, i.e., given $Z(T_A-)$ the jump $\Delta X(T_A)$ is independent of $Z(T_A - s)$ for any $s > 0$. Section 2 provides an example in which

$$X(T_A) = X(T_A-)\zeta \tag{51}$$

where ζ is independent and follows a lognormal. Another assumption (satisfied by the example in section 3) relates to a continuity property for the jump:

$$\lim_{s \to T_A} Z^{s,z}(T_A) = z + \Delta Z(T_A) \quad \text{a.s..} \tag{52}$$

We want to characterize the continuation region just before T_A and in particular we want to give sufficient conditions for the case when it is never optimal to stop just before the announcement. In the present context we need something similar to the Equation (38):

$$V_{T_A-} = \widetilde{E}_{(T_A)^-}[V_{T_A}].$$

Indeed we have the following:

Lemma 2 (L1) *Consider the model described in the present section. Assume further that condition C2 is true (see appendix 3A), that the value function V exists and that $V(T_A, z)$ is lower semi continuous in z. Then:*

$$\lim \inf_{t \to T_A-} V(t,z) \geq E\left[V(T_A, Z_{T_A})\,|Z_{T_A-} = z\right]. \tag{53}$$

The proof is technical and is left for the appendix A. The condition C2 guarantees that certain stopping times exists. This condition may hold quite generally but we were not able to prove it. The lower semi-continuity (l.s.c.) property isn't very restrictive. Indeed, as there are no jump after T_A, a sufficient condition is that g should be l.s.c. (see Oksendal (2003) Chapt. 10). The continuity property on the jump at T_A is quite general also.

3.1 Main Results

Here we characterize situations in which it is not optimal to stop just before the scheduled announcement. This is true if

$$\lim \inf_{t \to T_A} V(t,z) > g(z) \tag{54}$$

because in this case there is $\varepsilon > 0$ such that

$$V(t,z) > g(z) \quad \text{for } t \in (T_A - \varepsilon, T_A). \tag{55}$$

It is useful to define three regions. The first one is the set D_p where it is not optimal to stop at T_A with positive probability. In other word, z belongs to this set if the value function $V(T_A, Z_{T_A})$ is greater than $g(T_A, Z_{T_A})$ with positive probability.

Definition 3 *Define the set D_p as*

$$D_p = \left\{ z \in \Re^{n+m} | P\left[V(T_A, Z_{T_A}) > g(T_A, Z_{T_A}) | Z_{T_A-} = z\right] > 0 \right\}. \tag{56}$$

The other two sets relate only to the function g and the jump. For the elements in the set $D_>$ it is better to stop just after the announcement than just before (when comparing only those two options), i.e., for $z \in D_>$ we have that $E\left[g(T_A, Z_{T_A}) | Z_{T_A-} = z\right] > g(z)$. Similarly, for the element in $D_\geq$, the agent prefer to stop just after than just before or may be indifferent, i.e., for $z \in D_\geq$ we have that $E\left[g(T_A, Z_{T_A}) | Z_{T_A-} = z\right] \geq g(z)$. Those sets may be defined using the concept of certainty equivalence as well (note that the certainty equivalent state $c(z)$ is not unique in some cases).

Definition 4 *The certainty equivalent $c(z)$ is defined implicilty by the equation*

$$g(c(z)) = E\left[g(T_A, Z_{T_A}) | Z_{T_A-} = z\right]. \tag{57}$$

Definition 5 *Define the set $D_>$ as*

$$D_> = \left\{ z \in \Re^{n+m} | E\left[g(T_A, Z_{T_A}) | Z_{T_A-} = z\right] > g(z) \right\} \tag{58}$$

or, equivalently

$$D_> = \left\{ z \in \Re^{n+m} | g(c(z)) > g(z) \right\} \tag{59}$$

Definition 6 *Define the set $D_\geq$ as*

$$D_\geq = \left\{ z \in \Re^{n+m} | E\left[g(T_A, Z_{T_A}) | Z_{T_A-} = z\right] \geq g(z) \right\}. \tag{60}$$

or, equivalently

$$D_\geq = \left\{ z \in \Re^{n+m} | g(c(z)) \geq g(z) \right\}. \tag{61}$$

With those definition we can now enunciate the main proposition. It basically states that it is not optimal to stop just before the scheduled announcement in two situation: if the state variable z belongs to $D_>$ or if $z \in D_\geq \cap D_p$.

Proposition 7 *Consider the model defined in the present section and assume as true the hypothesis of lemma L1. Then, it is not optimal to stop just before the announcement if $z = Z(T_A-)$ belongs to $D_>$, i.e.:*

$$\lim_{t \to T_A} \inf V(t,z) > g(z) \quad \textit{for } z \in D_>. \tag{62}$$

Moreover if $Z(T_A-) = z \in D_\geq \cap D_p$ then it is not optimal to stop just before T_A, i.e.,

$$\lim_{t \to T_A} \inf V(t,z) > g(z) \quad \textit{for } z \in D_\geq \cap D_p. \tag{63}$$

Proof. It generalizes the same steps we did in the previous section:

$$\begin{aligned} \lim_{t \to T_A} \inf V(t,z) &\geq E\left[V(T_A, Z_{T_A}) | Z_{T_A-} = z\right] && (64)\\ &\geq E\left[g(Z_{T_A}) | Z_{T_A-} = z\right] && (65)\\ &\geq g(c(z)) && (66)\\ &\geq g(z). && (67) \end{aligned}$$

Then, for $z \in D_>$ the inequality in the last line is strict. Moreover, for $z \in D_p$ the inequality is strict in the second line. Finally, for both cases (i.e., for $z \in D_>$ and for $z \in D_\geq \cap D_p$):

$$\lim_{t \to T_A} \inf V(t,z) > g(z). \tag{68}$$

■

Recall that in order to define D_p we need to know the value function at T_A. However we can find a subset of D_p using only the model primitives and use this set instead of D_p in the above proposition.

Note that if $z \in D_p$ then $P\left[Z_{T_A} \in \mathbf{C} | Z_{T_A-} = z\right] > 0$ where $\mathbf{C} = \{(t,z) \in \Re \times \Re^{n+m} | V(t,z) > g(z)\}$ is the continuation region. The Proposition 2.3 in Oksendal and Sulem (2007) defines a subset of the continuation region using only the primitives of the model. Using this subset instead of $\mathbf{C}$ allows us to find a smaller set $U_p \subset D_p$ not using the value function at T_A.

Definition 8 *Define the set U_p as*

$$U_p = \left\{z \in \Re^{n+m} | P\left[Z_{T_A} \in U | Z_{T_A-} = z\right] > 0\right\}. \tag{69}$$

where

$$U = \left\{z \in \Re^{n+m} | Ag + f > 0\right\}$$

and A is the generator function associated to process Z_t.

In several situations the generator A may be replaced by the differential operator

$$Af(z) = \sum_i \alpha_i(z) \frac{\partial f}{\partial z_i}(z) + \sum \left(\sigma \sigma^T\right)_{ij} \frac{\partial^2 f}{\partial z_i \partial z_j}(z)$$

where σ^T is the transpose of σ. The next section provides an example. For details about the operator A we refer to Oksendal and Sulem (2007). With the set U we may establish the corollary:

Corollary 9 *Suppose the hypotheses of proposition above are satisfied. If $Z(T_A-) = z \in D_{\geq} \cap U_p$ then it is not optimal to stop just before T_A, i.e.,*

$$\liminf_{t \to T_A} V(t, z) > g(z) \quad \textit{for } z \in D_{\geq} \cap U_p. \tag{70}$$

In several cases, D_p or $D_>$ is all space (or both). It is true, for instance, if g is convex, the jump size expectation is zero ($E[Z_{T_A} | Z_{T_A-} = z] = z$) and it isn't optimal to exercise at T_A with positive probability. This is the case for American Options with convex payoff in the risk-neutral measure. Moreover, if $g(z) = g(y)$, i.e. if the payoff doesn't depends upon variables that jumps at T_A, then $D_{\geq}$ is all space.

Another interesting case is when g is CRRA (Constant Relative Risk Aversion):

$$g(x^0) = \frac{\left(x^0\right)^\gamma}{\gamma}. \tag{71}$$

where x^0 is a homogeneous scalar function of degree 1 in Z_t, $\gamma \in (0,1)$ (remember that $g(x^0) \geq 0$) and the jump at T_A is

$$x^0(Z_{T_A}) = x^0(Z_{T_A-})\xi \tag{72}$$

where ξ is independent of Z_{T_A-}. In this case, the certainty equivalent has a nice property. If

$$E\left[\frac{\left(x^0\right)^\gamma}{\gamma} | Z_{T_A-} = z\right] = \frac{c^\gamma}{\gamma} \tag{73}$$

then

$$E\left[\frac{\left(x^0\right)^\gamma}{\gamma} | Z_{T_A-} = 2z\right] = \frac{(2c)^\gamma}{\gamma}. \tag{74}$$

We sum up those observations in the following corollary:

Corollary 10 *Suppose the hypotheses of proposition above are satisfied. Then we have:*

(i) if g is increasing, convex and $E[Z_{T_A}] \geq Z_{T_A-}$ then it is not optimal to stop for $Z_{T_A-} = z \in D_p$;

(ii) If the payoff doesn't depends upon the variable that jumps, i.e., if $g(z) = g(x, y) = g(y)$ then it is not optimal to stop for $Z_{T_A-} = z \in D_p$;

(iii) If the payoff is a CRRA function, i.e., $g(z) = (x_0(z))^\gamma / \gamma$ where $x_0(z)$ is homogeneous scalar function of degree 1 in z, if the jump has the property that $x^0(Z_{T_A}) = x^0(Z_{T_A-})\xi$ and if $c(1) > 1$ then it is never optimal to stop just before T_A.

4 Another Application in Finance

TThe objective of the present section is twofold. First, it is an example of the above results. It applies the corollaries and defines the generator operator for one particular case. Second, it discusses a possible modeling for an agent who wants to sell an asset highlighting the incentives when there is a scheduled announcement. For the most part we explore the case in which the price doesn't jump with the announcements. It highlights some incentives and makes the results more clear. However in the last subsection we make comments on more general cases.

4.1 The Optimal Time to Sell with Transaction Cost

We will consider a problem of one agent (or investor) that wants to sell its portfolio and there is an information being released at a known date T_A. We are interested in his behavior around the date T_A. To be more clear, we want to show that selling just before T_A is less likely in some sense and may never be optimal in some circumstances. To simplify, we will consider that the portfolio has only one asset, the utility is linear and is obtained when the investor sells the portfolio at time τ:

$$J^{\tau}(x) = E^{s,x}\left[e^{-\rho\tau}\left(X(\tau) - a\right)\right] \tag{75}$$

where $X(t)$ is the price of the asset at time t, ρ is the discount factor, a is the fixed cost to sell the asset, $E^{s,x}[.]$ is the expectation operator conditional to information $\mathcal{F}_s$ obtained at s when $X(s) = x$, and τ is a stopping time.

The asset follows a Geometric Brownian Motion :

$$dX(t) = X(t^{-})\left[\alpha(t)dt + \beta dB(t)\right] \qquad X(s) = x > 0 \tag{76}$$

where $B(t)$ is the Wiener process, β and γ are constants, the function $\alpha(t)$ is constant before and after T. The impact of information on market is a random change on the coefficient $\alpha(t)$ at T_A. It is described as:

$$\alpha(t) = \alpha_0 \text{ if } t < T_A \tag{77}$$

$$\alpha(t) = \zeta \text{ if } t \geq T_A \tag{78}$$

where ζ is a random variable with uniform distribution in the interval $[\underline{\alpha}, \overline{\alpha})$ with $0 < \underline{\alpha} < \overline{\alpha} \leq \rho$, and $\alpha_0 < \rho$.

Note that for $\rho = \alpha$ we have the same problem as pricing American calls.

4.2 Solution Without Information Release

The problem without information release is the same as the example 2.5 of Oksendal and Sulem (2007). The only difference is that $\alpha(t) = \alpha_0$ for all t. We'll give the solution here because we will need it later.

Notice that it is never optimal to sell if $\rho < \alpha$ even if the cost a is zero (in this case $J^{\tau=\infty} = \infty$) and obviously it is never optimal to sell the asset if its price X is less than the cost a for any time (eventually the price will be more than a). We will call the continuation region $D_{noNews} \subset \Re^2$ as the set of time and prices that is not optimal to sell the asset (i.e. the continuation region). Oksendal and Sulem (2007) shows that:

$$\mathbf{C}_{noNews} = \{(s, x) : x < x^*\} \tag{79}$$

where x^* is defined below and doesn't depend upon time. This is consistent with the assertive that the problem faced by the agent at time s_1 with $X(s_1) = x$ is the same at time s_2 with $X(s_2) = X(s_1) = x$. The solution for $J^* = \sup_\tau J^\tau$ is:

$$J^*(s, x) = e^{-\rho s} C x^{\lambda_1} \quad \text{if } 0 < x < x^* \tag{80}$$

$$J^*(s, x) = e^{-\rho s}(x - a) \quad \text{if } x^* \leq x \tag{81}$$

where λ_1 is the solution of

$$0 = -\rho + \alpha\lambda_1 + \frac{1}{2}\beta\lambda_1(\lambda_1 - 1) \tag{82}$$

and

$$x^* = \frac{\lambda_1 a}{\lambda_1 - 1}, \tag{83}$$

$$C = \frac{1}{\lambda_1}(x^*)^{1-\lambda_1}. \tag{84}$$

Finally, if $\alpha = \rho$, it is never optimal to sell the asset and $J^*(s, x) = J^{\tau=\infty} = xe^{-\rho s}$.

4.3 When It Is Not Optimal to Sell Close to T

When $\alpha(t)$ changes randomly at T, the continuation region is no longer constant over time. Nonetheless for $s \geqslant T_A$ the optimization problem is the same as in the previous section and is never optimal to sell in the region:

$$\{(s, x, \alpha) : x < x^*(\alpha), s \geqslant T_A\}. \tag{85}$$

Notice that we add α to the notation. The solution is the same above:

$$J^*(s, x, \alpha) = e^{-\rho s} C(\alpha) x^{\lambda_1(\alpha)} \quad \text{if } 0 < x < x^*(\alpha) \text{ and } s \geqslant T_A \tag{86}$$

$$J^*(s, x, \alpha) = e^{-\rho s}(x - a) \quad \text{if } x^*(\alpha) \leq x \text{ and } s \geqslant T_A \tag{87}$$

where $\lambda_1(\alpha)$ is the solution of

$$0 = -\rho + \alpha\lambda(\alpha) + \frac{1}{2}\beta\lambda(\alpha)(\lambda(\alpha) - 1) \tag{88}$$

and

$$x^*(\alpha) = \frac{\lambda(\alpha)a}{\lambda(\alpha) - 1}, \tag{89}$$

$$C(\alpha) = \frac{1}{\lambda(\alpha)}(x^*)^{1-\lambda(\alpha)}. \tag{90}$$

4.3.1 And If the Solution After T_A Isn't Known?

In general the solution after T_A isn't known. On those cases it is possible to characterize subset of the inaction region (see Oksendal and Sulem (2007) for details). For this purpose, define the generator operator A as

$$Ag(s,x) = \frac{\partial g}{\partial s} + \alpha(s)x\frac{\partial g}{\partial x} + \frac{1}{2}\beta x^2 \frac{\partial^2 g}{\partial x^2}, \tag{91}$$

where $g = e^{-\rho\tau}(X(\tau) - a)$ and define the set U as:

$$U = \left\{(x,s,\alpha) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ | Ag + f > 0\right\} \tag{92}$$

where $f = 0$ in our problem. The proposition 2.3 in Oksendal and Sulem (2007) tell us that $U \subset \mathbf{C}$, i.e., it is never optimal to stop when $(x,s,\alpha) \in U$. We find that:

$$Ag + f = e^{-\rho s}((\alpha - \rho)x + \rho a) \tag{93}$$

and U is:

$$U_\alpha = \left\{(x,s,\alpha) | x < \frac{\rho a}{\rho - \alpha}\right\}. \tag{94}$$

Realize that if $\alpha(s) = \rho$, the continuation region after T is:

$$\{(s,x) : x < \infty, s > T_A\}. \tag{95}$$

4.4 Numerical Solution

4.4.1 Algorithm Overview

Oksendal and Sulem (2007) provide a sufficient conditions for a function to be a solution of the above problem. Those conditions are called integrovariational inequalities for optimal stopping time and are characterized by the formulas

$$\max(A\phi, g - \phi) = 0 \tag{96}$$

$$\mathbf{C} = \left\{(s,x) \in R^+ \times R^+ | \phi(s,x) > g(s,x)\right\} \tag{97}$$

along with regularity conditions, where A is defined as above. Realize that the problem isn't only to find ϕ, but to find the region $\mathbf{C}$ as well, i.e., finding the right boundary conditions is part of the problem .

We want to solve it numerically using some kind of finite difference approximation for the operator A. Nonetheless, the usual methods cannot be applied directly because the boundary conditions aren't defined from the outset. In pricing American Options, it is common to overcome this difficult using the so called Projected Successive Over Relaxation, a generalization of the Gauss–Seidel method. Nonetheless, we will use a policy iteration algorithm provided by Chancelier et al. (2007). We detail the method in appendix B but we give an overview here.

In our case this is done by considering a rectangular grid. The equation above is rewritten as $\max\left(A_h\phi_h, g_h - \phi_h\right) = 0$ and $\mathbf{C}_h = \{(s,x) \text{ belongs to grid} | \phi_h(s,x) > g_h(s,x)\}$ [17].This problem is equivalent to a better behaved one, defined as:

$$\phi_h = \max\left(\left[I_h + \frac{\xi A_h}{1+\xi\rho}\right]\phi_h, g_h\right) \tag{98}$$

where $0 < \xi \leq \min \frac{1}{|(A_h)_{ii}+\rho|}$, and I_δ is the identity operator ($I_h v_h = v_h$). The solution is found iteratively: in the first iteration, define D_h^1 and solve $\frac{\xi A_h}{1+\xi\rho}\phi_h^1 = 0$ for $(s,x) \in \mathbf{C}_h^1$ defining $\phi_h^1 = g_h(s,x)$ for $(s,x) \notin \mathbf{C}_h^1$. In the second iteration, define D_h^2 as the points in the grid that $\left(I_h + \frac{\xi A_h}{1+\xi\rho}\right)\phi_h^1 > g_h(s,x)$, then solve $\frac{\xi A_h}{1+\xi\rho}\phi_h^2 = 0$ for $(s,x) \in \mathbf{C}_h^2$ defining $\phi_h^2 = g_h(s,x)$ for $(s,x) \notin \mathbf{C}_h^2$. Keep iterating until it converges. Chancelier et. al. (2007) shows that this procedure converges to the right solution.

For $s < T_A$ we assume that

$$\lim_{s\to T_A-}\phi_h(s,x) = E\left[\phi_h(T_A,x)\right]. \tag{99}$$

We don't prove this statement but lemma L1 implies that $\lim_{s\to T_A-}\phi_h(s,x) \geq E\left[\phi_h(T_A,x)\right]$. Then we are assuming a lower bound if the equality in equation (99) does not hold. In this case the numerical solution would have a downward bias when compared to the true solution. This bias lead to a smaller continuation before the announcement. As some of ours analysis are based on how big is $\mathbf{C}$ before the announcement our results are conservative.

4.4.2 The Results for Two Different Simulations

Solution is found for two configurations of parameters (see table table 1). Notice that the only differences in the two cases are the parameters $\underline{\alpha}$.

The figure 1 shows the region $\mathbf{C}^1$. It is interesting to compare $\mathbf{C}^1$ with the continuation region $\mathbf{C}^{noNews}$ for the problem without information release and the same parameters. To this end a dashed horizontal line at price $x^*(\alpha = 0.1) = 104.24$ represents the upper boundary of $\mathbf{C}^{noNews}$. We can separate three interesting regions in the time. When the information is far (in our case,

[17] The subscript δ denotes the approximation of functions or operators defined on the grid.

Table 1: Two Parameters Configurations.

Parameter	Case 1	Case 2
α	0.1	0.1
σ	0.4	0.4
ρ	0.12	0.12
a	10	10
T	10	10
$\underline{\alpha}$	0	0.095
$\overline{\alpha}$	0.11	0.11

for $t = 0$) $\mathbf{C}^1$ is similar to $\mathbf{C}^{noNews}$, but lays a little below. Then, $\mathbf{C}^1$ make an U shape and finally increases getting close to price $x^*(\overline{\alpha} = .11) = 204.1211$ at the time T_A. The figure 2 shows the difference between the value functions for parameter in case 1 (table 1) and for the model without information release with contour curves[18] for $z = V_1 - V_{noNews}$. For $z > 0$ it means that $V_1 > V_{noNews}$ and it happen only at a small region close to T_A. For the most part $z = 0$ or $z < 0$.

For the most part of time the agent isn't better off when compared to the case without announcement. This is explained by the choice of the parameter $\underline{\alpha}$ as zero. In this case, it is much more likely that the parameter α_T will be less than α by a good amount[19], making the agent worse off. This effect is damped when the announcement is far because it is more likely to sell the asset before T. When the time is close to the announcement the agent will probably sell the asset in an adverse environment because α_T will probably be lower. Nonetheless, when the price is "high" (i.e. the price is close to the boundary of D^{sim1}) for a time close to the news, others incentives enter into play. In this case, the agent would sell the asset for this "high" price but can wait a little to see if the realization of α_T makes him better off. In a good realization, the agent probably will "make some money" taking more time to sell the asset. In a bad realization the investor sells it right away, and the "loss" taken to wait a little is probably small. In other words, on those situation, it is worth to wait a little for more information.

The figures 3 and 4 are of the same type as figures 1 and 2, respectively. The odds now are in favour to make α_T higher than α in a good amount. The agent now is always better off when compared with the case without information release. Realize that $\mathbf{C}^{noNews} \subset \mathbf{C}^1$ and that the boundary increases monotonically with time until T_A. When the news is far from being released, $\mathbf{C}^1$ is similar to $\mathbf{C}^{noNews}$ and value function is just a little bit higher. For "high " prices it may be worth to wait a little more as the incentive to sell is weakened. As the announcement gets closer, the possibility of sell at even higher prices if $\alpha_T > \alpha$ makes the continuation region get wider at a faster pace.

[18] A contour line (also isoline) of a function of two variables is a curve along which the function has a constant value.

[19] A good amount, when compared to the possibles α_T higher than α.

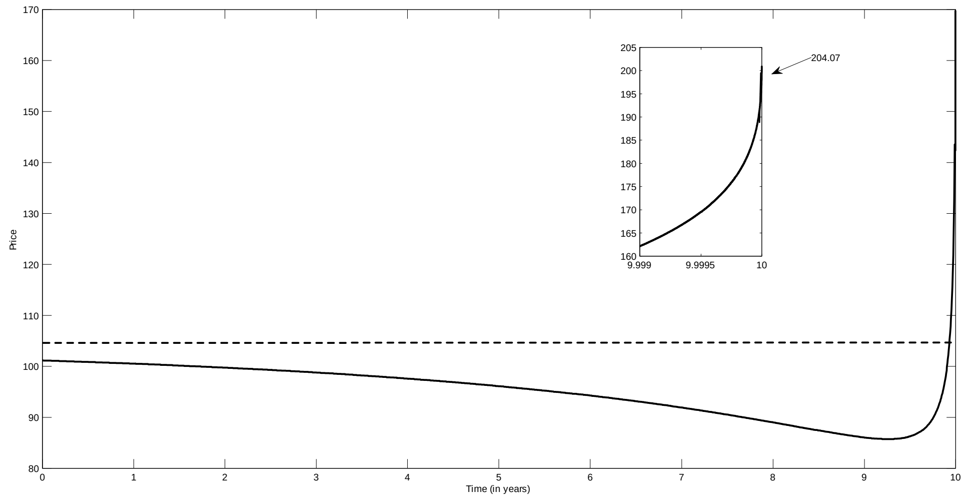

Figure 1: The figure shows the continuation regions for the parameters in table 1, case 1. The solid line and the dashed line represents the upper boundary of $\mathbf{C}^1$ and $\mathbf{C}^{noNews}$ respectively. The inside graph shows a more detailed simulation close to the announcement.

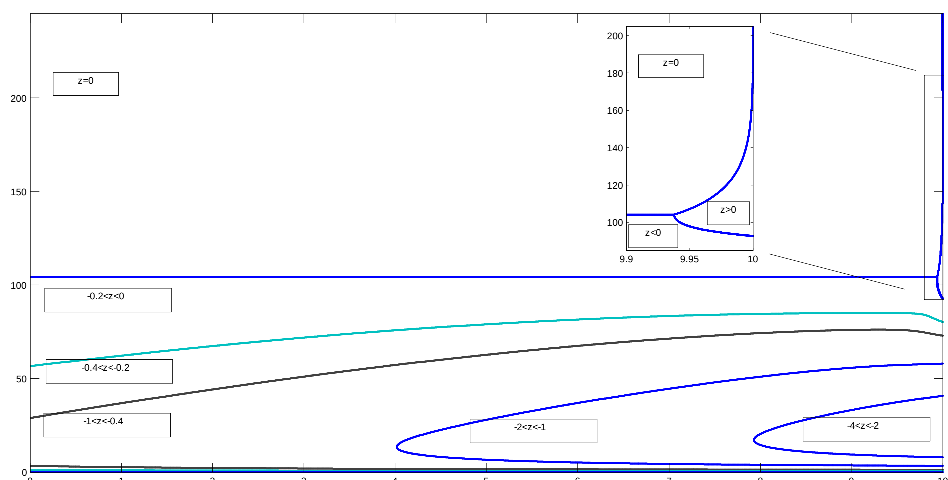

Figure 2: Contour line (or isoline) for $z = V_1 - V_{noNews}$. Realize that z is greater than zero only in a small region.

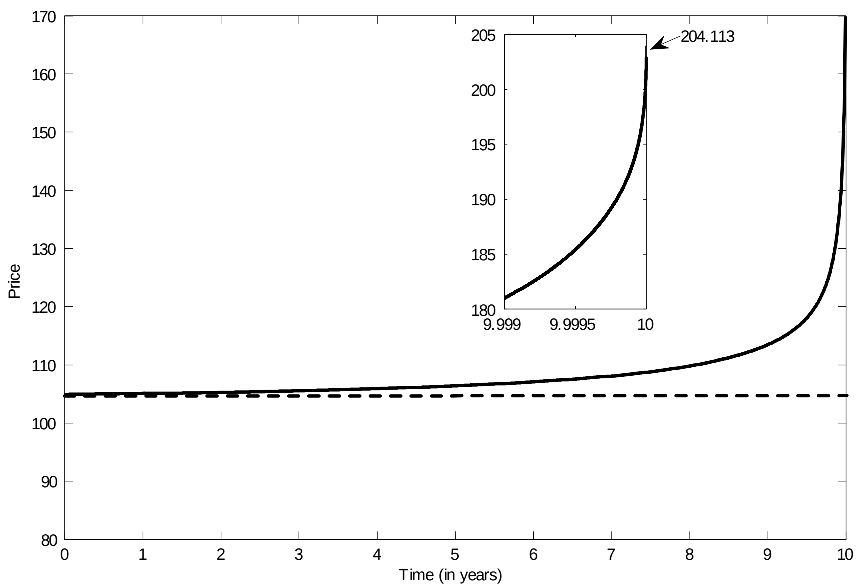

Figure 3: Continuation regions for the numerical solution for parameters in case 2, table 1. The solid line and the dashed line represents the upper boundary of $\mathbf{C}^2$ and $\mathbf{C}^{noNews}$ respectively. The inside graph shows a more detailed simulation close to the announcement.

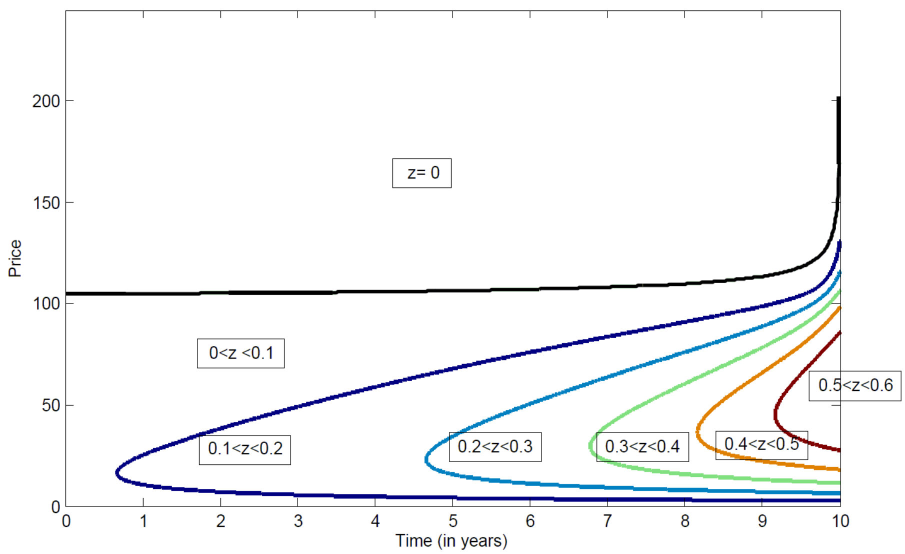

Figure 4: Contour line (or isoline) for $z = V_2 - V_{noNews}$. Realize that $z \geq 0$ in all region.

In both cases, the boundary of continuation region increases and gets close to $x^*(\overline{\alpha} = .11) = 204.1211$ as the time gets close to T_A.

4.5 Discretionary Liquidity Traders

This behavior illustrates the incentive the DLTs face when trying to sell an asset given that she knows the price won't jump but the process will change somehow. Those simplification has the purpose of intepret some incentives avoiding the analysis of the effect of jumps. In this case, the main benefit is to wait a little more and sell for a better price. If the news affects negatively the trend of the price, usually it is better to sells immediately after the news. As we are not considering that a jump may occur in the price due the announcement, it doesn't hurt the agent to wait a little. If we can summarize the result in one statement, it would be that the agent prefers to sell with more information as long as wait for such thing has low a risk.

We considered a special case where price doesn't jump and the agent is risk-neutral. More generally the results applies if the agent has a CRRA utility function and the price jumps with positive average (big enough to account for risk aversion). It is interesting to mention that Bamber et al. (1998) finds that only one quarter of the price had a sudden impact on prices. Then it is probable the DLTs are in a situation between the no jump and the case with a positive average jump.

4.6 Interpretations

This behavior illustrates the incentive an agent face when trying to sell an asset given that he/she knows the price won't jump but the process will change somehow. That simplification has the purpose of intepret some incentives avoiding the analysis of the effect of jumps. In this case, the main benefit is to wait a little more and sell for a better price. If the news affects negatively the trend of the price, usually it is better to sells immediately after the news. If we can summarize the result in one statement, it would be that the agent prefers to sell with more information as long as waiting for such thing has low a risk.

We considered a special case where price doesn't jump and the agent is risk-neutral. More generally the results applies if the agent has a CRRA utility function and the price jumps with positive average (big enough to account for risk aversion). It is interesting to mention that Bamber et al. (1998) finds that only at one quarter of time the prices had a sudden impact. Then it is probable the investors are in a situation between the no jump and the case with a positive average jump.

5 Discussion

Under mild conditions, optimal stopping time problems entail a time and state dependent rule: it is optimal to stop whenever the process goes out the continuation region. It implies a higher chance to stop at jumps regardless it happens at fixed or random times. On the other hand, it is harder going out the continuation region when it is bigger (in general) and the main message of the present work is that it is indeed bigger just before a fixed jump for some common situations. In other words, it is less probable to stop before a fixed (and known) jump time when compared to "normal" times for some common cases. Moreover, it is possible to predict this behavior without solving the problem in some cases by applying the generator operator to the reward function.

Such time state dependent rules may arise in several economic situations. For instance it is true in resetting price models with menu cost or optimal portfolio problems with fixed cost. Although those problems may be considered as a sequence of optimal stopping time, we are analyzing here the simplest case of single stopping. This might be a good way to model agents who wants to sell an asset (such as a house or a stock) specially in the presence of fixed cost.

Based on empirical evidence, it is reasonable to assume that prices jump (with positive probability) when relevant information hits the markets. It is true for corporate or market events containing relevant information whether it is a scheduled one or not. Then any investor with state dependent strategy has a higher chance to trade at those times or a little after. This might be an important piece in the explanation of higher volume after announcements. Note that there is no need to incorporate information asymmetries or difference in opinion to obtain the time and state dependent rules. Those considerations are also valid for the decrease in volume before the scheduled announcements, especially in the presence of the type of investor analyzed here. They may prefer to trade only after the announcement even if there is no asymmetry or no chance to engage in an adverse transaction before the event with a more informed investor. Another possible incentive is the average positive price change as is documented in the earning announcement premium (see, for instance, Frazzini and Lamont (2007) or Barber *et al.* (2013)).

We focus on the price as the important state because its role and behavior are clearly observed. Nonetheless other state variable may be considered as well. Some investor may focus their strategies on some fundamental signal such as book-to-value or price-to-earnings. It is even possible to consider some qualitative state such as belonging to an index or the existence of some legal issue. Then, even without change in prices, announcements might spur trades after and decrease volume before it.

6 Conclusion

In the present work we investigate the optimal stopping time in continuous time models when there is a jump at a fixed and known date. We characterize the continuation region a little before the jump showing that it is better not to stop just before the news in several situations of interest. Moreover in order to verify such characteristic in a model one needs only to apply the generator operator to the reward function without solving the problem.

These results are used to analyze some financial situations as empirical findings suggest that the price jump with positive probability at scheduled announcement. American Options are modeled as an optimal stopping time problem and we show that if the payoff is convex then it is never optimal to exercise just before the announcement. Moreover, we want to add some theoretical observations about the behavior of the volume around the announcements. Several authors stress out the role of agents with exogenous reasons for sell an asset and we model these investors as facing an optimal stopping time problem. Using the general results we argue that such investors may prefer to transact after the announcements. It happens because the agent "wants" to know the changes caused by the announcement and because the agent "wants" to gather the positive premium usually associated with announcements (such as the earnings announcements premium). Moreover we give the numerical solution for the case of a risk neutral investor facing a fixed costs and use a relatively recent numerical method.

Much of the intuition comes from the time and state dependent strategy implied by the optimal stopping times solution. Such strategies are pervasive in economic especially in situations where some sort of cost (e.g., fixed cost) exist. For instance, a portfolio problem similar to Merton (1969) but with fixed cost imply an optimal impulse problem combined with optimal stochastic problem. To analyze those type of problems when there are a jump at fixed and known date are subject of future research.

A Precise Definitions and Proofs

The objective of the present appendix is to define precisely the elements of section 3 and extend it to the jump-diffusion case. The definitions are quite general but we make clear what assumption is being used. In particular we make precise the general condition the jump at the announcement (time T_A) should satisfy.

The first step towards proving lemma L1 is to show an inequality on $V(t, Z_t)$ where $V(t, z)$ is the Value Function. This inequality is similar to the property of supermartingales. Note that Z_t is the solution of a stochastic differential equation (SDE) and a more complete notation would be $Z_t^{s,z}$ where the superscript s, z means that $Z_t^{s,z}$ is the value of the process at t with the initial condition $Z(s) = z$.

Finally we make the assumption that $V(t, z)$ is lower semi-continuous (l.s.c.) in z for $t = T_A$ and that the jump at T_A has some continuity properties. Npte that the lower semi-continuity property isn't very restrictive. For instance, if g is l.s.c. and the process Z_t has no jump after T_A then $V(t, z)$ is l.s.c. for $t \geq T_A$ (see Oksendal (2003) Chapt. 10). The continuity property on the jump at T_A is quite general also.

A.1 Definitions

Consider the probability space $(\Omega, \mathcal{F}, P)$ and the filtration $\mathcal{F}_t$. Fix an open set $S \subset \mathbb{R}^{n+m}$ (the solvency region) and let $Z(t)$ be a jump diffusion cadlag process in $\mathbb{R}^{n+m}$ given by

$$dZ(t) = \alpha(Z(t))dt + \sigma(Z(t))dB(t) + \int_{\mathbb{R}^{n+m}} \gamma(Z(t^-), z')\widetilde{N}(dt, dz'), \tag{100}$$

$$Z(s) = z \in \mathbb{R}^{n+m}, \tag{101}$$

where $b(.)$, $\sigma(.)$ and $\gamma(.)$ are functions such that a unique solution to $Z(t)$ exists (see Oksendal and Sulem (2007), Theorem 1.19), $B(t)$ is the n+m dimensional Wiener process and $\widetilde{N}$ is the compensated Poisson random measure.

The integral incorporates jumps into the process. In order to define the compensated Poisson random measure completely, we define the Poisson random measure $N(t, U)$ as the number of jumps of size $\Delta Z \in U$ (where U is a borel set whose closure doesn't contain the origin) which occur before or at time t. We need the Levy measure also:

$$\nu(U) = E[N(1, U)] \tag{102}$$

where U is a borel set whose closure does not contain the origin. There is $R \in [0, \infty]$ where

$$\begin{aligned} \widetilde{N}(dt, dz) &= N(dt, dz) - \nu(dz)\,dt \quad \text{if } |z| < R && (103) \\ &= N(dt, dz) \quad \text{if } |z| \geq R && (104) \end{aligned}$$

and z is inside the integrand. For more details we refer to Protter (2003) and Oksendal and Sulem (2007).

The process Z_t (recall that $Z_t = Z(t)$) is divided in two process: $X_t \in \Re^m$ that jump with positive probability at T_A; and $Y_t \in \Re^n$ that doesn't jump at T_A almost surelly

$$Z(t) = (Y(t), X(t)), \tag{105}$$

$$Y(T_A) = Y(T_A-) \text{ a.s.} \tag{106}$$

$$X(T_A) = X(T_A-) + \Delta X(T_A), \tag{107}$$

where $\Delta X(T_A) \neq 0$ with positive probability and $\Delta X(T_A)$ is $\mathcal{F}_{T_A}-$measurable random variable. A more complete notation is $Z^{s,z}(t)$ indicating that it is a solution of the SDE in equation (100) with the initial condition $Z(s) = z$, i.e.,

$$Z^{s,y}(t) = z + \int_s^t \alpha(Z(t))du + \int_s^t \sigma(Z(u))dB(u) + \int_s^t \int_{\mathbb{R}^{n+m}} \gamma(Z(u^-), z')\widetilde{N}(du, dz'). \tag{108}$$

The expecation operator $E^{s,z}[h(Z_t)]$ is defined as[20]

$$E^{s,z}[h(Z_t)] = E[h(Z_t^{s,z})]. \tag{109}$$

We make the assumption that the random variable $\Delta X(T_A)$ depends only upon $Z(T_A-)$, i.e., given $Z(T_A-)$ the jump $\Delta X(T_A)$ is independent of $Z(T_A - s)$ for any $s > 0$. Section 2 provides an example in which

$$X(T_A) = X(T_A-)\zeta \tag{110}$$

[20]We will write $E^{y,s}[h(Y(t))]$ and $E[h(Y^{s,y}(t))]$ interchangeably.
I'm following the notation used in Shreve (2004), Stochastic Calculus for Finance II. This expectation is defined on chapter 6, page 266.

where ζ is independent from $X(t)$ for $t < s$ and that the conditional distribution is lognormal. Another assumption (satisfied by the example in section 3) relates to a continuity property:

$$\lim_{s \to T_A} Z^{s,z}(T_A) = z + \Delta Z(T_A) \quad \text{a.s..} \tag{111}$$

Let

$$\tau_{s,z}^{S} = \inf\{t > s | Z^{s,y}(t) \notin S\}. \tag{112}$$

For notation sake, τ_S will be used instead of $\tau_{s,z}^S$ whenever it is clear which (s,z) is the right one. For instance $E^{s,z}[h(\tau_S)] = E^{s,z}\left[h\left(\tau_{s,z}^S\right)\right]$ unless state otherwise explicitly.

Let $f : \mathbb{R}^{n+n} \to \mathbb{R}$ and $g : \mathbb{R}^{n+m} \to \mathbb{R}$ be continuous functions satisfying the conditions:

$$E^{s,z}\left[\int_s^{\tau_S} f(Y(t^-))dt\right] < \infty \text{ for all } z \in \mathbb{R}^{n+m} \text{ and } s \geq 0 \tag{113}$$

and assume that the family $\left\{g(Z(\tau^-))\chi_{\{\tau<\infty\}}\right\}$ is uniformly integrable for all $z \in \mathbb{R}^{n+m}$, where $\chi_{\{.\}}$ is the indicator function and $f(Y(t-)) = \lim_{s \to t-} f((Y(s))$. We assume further that $f \geq 0$ and $g \geq 0$.

Let $\Upsilon^{s,z}$ be the set of of all optimal time $s \leq \tau \leq \tau_{s,z}^S$ and define the utility (or performance) function as

$$J^{\tau}(s,z) = E^{s,z}\left[\left(\int_s^{\tau} f((Z(t))dt + g(Z(\tau))\chi_{\{\tau<\infty\}}\right)\chi_{\{\tau \geq s\}}\right]. \tag{114}$$

The general optimal stopping problem is to find the supremum:

$$V(s,x) = \sup_{\tau \in \Upsilon_s^{s,y}} J^{\tau}(s,z), \ z \in \mathbb{R}^{n+m}. \tag{115}$$

Note that for $s \geq T_A$ we have the same situation as in Oksendal and Sulem (2007), chapter 2, and if there is no jump, it is the same as in Oksendal (2003), chapter 10, and all results therein applies.

A.2 Proof for lemma L1

It is important to emphasize the assumption about the limiting behavior:

Condition 11 (C1) *The jump at T_A has the limiting behavior*

$$\lim_{s \to T_A} Z^{s,z}(T_A) = z + \Delta Z(T_A) \quad \textit{a.s..} \tag{116}$$

We need another condition relating the utility function at two different times. For instance, we want to compare J^{τ_1} at s and something like J^{τ_2} at t for $s < t$. However there are some details in how to compare τ_1 and τ_2 as each one belongs to different sets: Υ^{s,z_1} and Υ^{t,z_2} respectively. Another difficulty in the definitions lies on how to relate z_1 and z_2. We solve it by considering $z_1 = z$ and $z_2 = Z_t^{s,z}$ and, in turn, the sets $\Upsilon^{s,z}$ and $\Upsilon^{t,Z_t^{s,z}}$. In this case the stopping time τ_2 may depend upon $Z_t^{s,z}$. In order to obtain our results we conjecture that the following is true:

Condition 12 (C2) *Let $\tau_2(Z_t^{s,z}) \in \Upsilon^{t,Z_t^{s,z}}$. For $s<t$, there is $\tau_1 \in \Upsilon^{s,z}$ such that:*

$$E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] \geq E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\left(J^{\tau_2(Z_t^{s,z})}(t,Z_t^{s,z})\right)\right]. \tag{117}$$

where $a \wedge b = \min(a,b)$,

Given the condition C2 (and that $f \geq 0$) we obtain an inequality for $\chi_{\{\tau_{s,z}^S \geq s\}} V\left(t, Z_t^{s,z}\right)$ that is important to what follows:

Lemma 13 *Consider the model defined in the first section of this appendix. If condition C2 holds then we have for $s<t$*

$$V(s,z) \geq E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}} V\left(t, Z_t^{s,z}\right)\right] \tag{118}$$

Proof. There are two cases: $V\left(t, Z_t^{s,z}(\omega)\right) < \infty$ a.s. and $V\left(t, Z_t^{s,z}\right) = \infty$ with positive probability (where $\omega \in \Omega$).

- Case 1: $V\left(t, Z_t^{s,z}(\omega)\right) < \infty$ a.s.:

As $V\left(t, Z_t^{s,z}(\omega)\right) < \infty$ a.s., for each $\varepsilon > 0$ there is $\tau_2\left(Z_t^{s,z}(\omega)\right) \in \Upsilon^{t,Z_t^{s,z}(\omega)}$ with the property

$$J^{\tau_2}(t, Z_t^{s,z}(\omega)) > V\left(t, Z_t^{s,z}(\omega)\right) - \varepsilon. \tag{119}$$

and

$$E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}} J^{\tau_2}(t, Z_t^{s,z})\right] > E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}} V\left(t, Z_t^{s,z}\right)\right] - \varepsilon E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\right] \tag{120}$$

Condition C2 guarantees that for each $\tau_2 = \tau_2(Z_t^{s,z}) \in \Upsilon^{t,Z_t^{s,z}}$ there is $\tau_1 \in \Upsilon^{s,z}$ such that:

$$E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] \geq E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}} J^{\tau_2(Z_t^{s,z})}(t,Z_t^{s,z})\right], \tag{121}$$

this implies:

$$E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] > E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}} V\left(t, Z_t^{s,z}\right)\right] - \varepsilon E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\right]$$

then

$$\begin{aligned} &\sup_{\tau_1 \in \Upsilon^{s,z}} E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] \\ > \;& E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}} V\left(t, Z_t^{s,z}\right)\right] - \varepsilon E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\right]. \end{aligned} \tag{122}$$

As this is true for all $\varepsilon > 0$ we have that

$$\sup_{\tau_1 \in \Upsilon^{s,z}} E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] \geq E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}} V\left(t, Z_t^{s,z}\right)\right]. \tag{123}$$

Now we need to show that $V(s,z)$ is greater than or equal to the l.h.s. in the above equation.

Note that for any $\tau_1 \in \Upsilon^{s,z}$ we have

$$\begin{aligned}
V(s,z) &\geq E^{s,z}\left[\int_s^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right] \\
&\geq E^{s,z}\left[\chi_{\{\tau^S_{s,z}\geq t\}}\left(\int_s^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] \\
&\geq E^{s,z}\left[\chi_{\{\tau^S_{s,z}\geq t\}}\left(\int_s^{t\wedge\tau_1} f(Z^{t,z}(t))dt + \int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] \\
&\geq E^{s,z}\left[\chi_{\{\tau^S_{s,z}\geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right]
\end{aligned}$$

where the last inequality is true because $\int_s^{t\wedge\tau_1} f(Z^{t,z}(t))dt \geq 0$ a.s.. As it is valid for any $\tau_1 \in \Upsilon^{s,z}$, it is valid also for the supremum:

$$V(s,z) \geq \sup_{\tau_1\in\Upsilon^{s,z}} E^{s,z}\left[\chi_{\{\tau^S_{s,z}\geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right]$$

and comparing with inequality 123 we have finally

$$V(s,z) \geq E^{s,z}\left[\chi_{\{\tau^S_{s,z}\geq t\}} V(t, Z_t^{s,z})\right].$$

- Case 2: $V(t, Z_t^{s,z}) = \infty$ with positive probability

For ω in which $V(t, Z_t^{s,z}(\omega)) = \infty$ we have that for $k > 0$ there is $\tau_2(Z_t^{s,z}) \in \Upsilon^{t,Z_t^{s,z}}$ such that

$$J^{\tau_{2(\omega)}}(t, Z_t^{s,z}(\omega)) > k. \tag{124}$$

and for ω in which $V(t, Z_t^{s,z}(\omega)) < \infty$ we have $\tau_{2(\omega)} \in \Upsilon^{t,Z_t^{s,z}}$ such that

$$J^{\tau_{2(\omega)}}(t, Z_t^{s,z}(\omega)) > V(t, Z_t^{s,z}(\omega)) - \varepsilon. \tag{125}$$

By condition C2 we can make

$$E^{s,z}\left[\chi_{\{\tau^S_{s,z}\geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] \geq E^{s,z}\left[\chi_{\{\tau^S_{s,z}\geq t\}}\left(J^{\tau_2(Z_t^{s,z})}(t, Z_t^{s,z})\right)\right],$$

and

$$E^{s,z}\left[\chi_{\{\tau^S_{s,z}\geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] > kE^{s,z}\left[\chi_{\{\tau^S_{s,z}\geq t\}}\chi_{\{V(t,Z_t^{s,z})=\infty\}}\right].$$

This is possible to make for all $k > 0$. If $E^{s,z}\left[\chi_{\{\tau^S_{s,z}\geq t\}}\chi_{\{V(t,Z_t^{s,z})=\infty\}}\right] > 0$ then we have

$$V(t,z) = \infty. \tag{126}$$

On the other hand, if $E^{s,z}\left[\chi_{\{\tau^S_{s,z}\geq t\}}\chi_{\{V\left(t,Z^{s,z}_t\right)=\infty\}}\right]=0$,

$$E^{s,z}\left[\chi_{\{\tau^S_{s,z}\geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt+g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right]\geq E^{s,z}\left[\chi_{\{\tau^S_{s,z}\geq t\}}V\left(t,Z^{s,z}_t\left(\omega\right)\right)\right]-\varepsilon E^{s,z}\left[\chi_{\{\tau^S_{s,z}\geq t\}}\right]$$

and the arguments of the case 1 applies. ■

Now we generalize the lemma L1 to the the jump-diffusion case. First we prove a statement using a sequence of time converging to T_A.

Lemma 14 *Assume as true the conditions in the previous proposition and that $V\left(T_A,z\right)$ is measurable in z. Then for any sequence $\{u_i\}_{i=1}^{\infty}$ such that $u_i<T$ and $\lim u_i=T$:*

$$\liminf_{i\to\infty} V\left(u_i,z\right)\geq E\left[\liminf_{i\to\infty} V\left(T_A,Z^{u_i,z}(T_A)\right)\chi_{\{\tau^S_{u_i,z}\geq T_A\}}\right]. \tag{127}$$

Proof. Using the lemma above:

$$V\left(u,z\right)\geq E^{u,z}\left[V\left(T_A,Z_{T_A}\right)\chi_{\{\tau_S\geq T_A\}}\right]. \tag{128}$$

Remember that

$$E^{u,z}\left[V\left(T_A,Z_{T_A}\right)\chi_{\{\tau_S>T_A\}}\right]=E\left[V\left(T_A,Z^{u,z}_{T_A}\right)\chi_{\{\tau_S\geq T_A\}}\right]. \tag{129}$$

As the inequality (128) is valid for all $0\leq u<T_A$, we have that:

$$\liminf_{i\to\infty} V\left(u_i,z\right)\geq \liminf_{i\to\infty} E\left[V\left(T_A,Z^{u_i,z}_{T_A}\right)\chi_{\{\tau_S\geq T_A\}}\right] \tag{130}$$

We want to use Fatou's lemma in the next step. Then we need to verify that $V\left(T_A,Z^{u_i,z}_{T_A}\right)\chi_{\{\tau_S\geq T_A\}}\geq 0$ a.s. and that it is measurable. As $f\geq 0$ and $g\geq 0$, we have that $V\left(T_A,Z^{u_i,z}_{T_A}\right)\chi_{\{\tau>T\}}\geq 0$. Moreover, $V\left(T_A,Z^{u_i,z}_{T_A}\right)\chi_{\{\tau_S>T\}}$ is $\mathcal{F}_{T_A}-$measurable random variable as it is a compositions of a measurable function $V\left(T_A,\cdot\right)$ with a $\mathcal{F}_{T_A}-$measurable random variable $Z^{u_i,z}_{T_A}$. Then, for any sequence $\{u_i\}_{i=1}^{\infty}$ such that $u_i<T_A$ and $\lim u_i=T_A$ we have that:

$$\liminf_{i\to\infty} V\left(u_i,z\right)\geq E\left[\liminf_{i\to\infty} V\left(T_A,Z^{u_i,z}(T_A)\right)\chi_{\{\tau_S\geq T_A\}}\right]. \tag{131}$$

■

The next two lemmas are similar to the lemma L1 in section 3. The statement explicitly mentions the solvency region. In the first version of the lemma the solvency region is all the space as is implicitly assumed in section 3. In the second version the solvency region may be any open set constant through time.

Lemma 15 (L1') *Consider the model defined in first section of this appendix and assume the conditions C1 and C2 as valid. Moreover assume that* $V(T_A, z)\,\chi_{\{\tau_S \geq T_A\}}$ *is* $\mathcal{F}_{T_A}-$*measurable and lower semi-continuous in* z *and that the solvency region* S *is all space. Then:*

$$\lim_{s \to T_A -}\inf V(s, z) \geq E\left[V\left(T_A, z + \Delta Z(T_A, z)\right)\right] \tag{132}$$

Proof. By condition C1 we have

$$\lim_{s \to T_A} Z^{s,z}(T_A)(\omega) = z + \Delta Z(T_A)(\omega) \quad \text{a.s.}. \tag{133}$$

Then, by properties of l.s.c. function (and noting that $\chi_{\{\tau_S \geq T_A\}} = 1$ because the solvency region is all space) , we have

$$\begin{aligned} \lim_{i \to \infty}\inf V\left(T_A, Z^{u_i,z}(T_A)(\omega)\right)\chi_{\{\tau_S \geq T_A\}} &\geq V\left(T_A, \lim_{i \to \infty}\inf Z^{u_i,z}(T_A)(\omega)\right) * 1 \\ &\geq V\left(T_A, z + \Delta Z(T_A)(\omega)\right) \end{aligned} \tag{134}$$

or

$$\lim_{i \to \infty}\inf V\left(T_A, Z^{u_i,z}(T_A)\right) \geq V\left(T_A, z + \Delta Z(T_A)\right) \quad \text{a.s.} \tag{135}$$

Then, the previous lemma implies that

$$\begin{aligned} \lim_{i \to \infty}\inf V(u_i, z) &\geq E\left[\lim_{i \to \infty}\inf V\left(T_A, Z^{u_i,z}(T_A)\right)\chi_{\{\tau_S \geq T_A\}}\right] \\ &\geq E\left[\lim_{i \to \infty}\inf V\left(T_A, Z^{u_i,z}(T_A)\right)\right] \\ &\geq E\left[V\left(T_A, z + \Delta Z(T_A)\right)\right] \end{aligned} \tag{136}$$

as this inequality is valid for all sequence $\{u_i\}$ converging to the announcement time $\lim_i u_i = T_A$,then it is also valid for the time limit $\lim s = T_A$.

$$\lim_{s \to T_A -}\inf V(s, z) \geq E\left[V\left(T_A, z + \Delta Z(T_A)\right)\right]. \tag{137}$$

■

Lemma 16 (L1") *Consider the model defined in first section of this appendix and assume the conditions C1 and C2 as valid. Assume that* $V(T_A, z)\,\chi_{\{\tau_S \geq T_A\}}$ *is* $\mathcal{F}_{T_A}-$*measurable and lower semi-continuous in* z*. Moreover, assume that the solvency region* S *doesn"t depend upon time. Then for* $z \in S$ *or* $z \notin \overline{S}$ *(where* $\overline{S}$ *is the closure of* S*) we have*

$$\lim_{s \to T_A -}\inf V(s, z) \geq E\left[V\left(T_A, z + \Delta Z(T_A, z)\right)\chi_{\{z \in S\}}\right] \tag{138}$$

Proof. If $z \in S$ (recall that S is an open set), then for all ω such that

$$\lim_{s \to T_A} Z^{s,z}(T_A)(\omega) = z + \Delta Z(T_A)(\omega) \tag{139}$$

there is s^* such that

$$Z^{s^*,z}(t) \in S \text{ for } s^* \leq t < T_A. \tag{140}$$

In this case

$$\begin{aligned} \liminf_{i \to \infty} V(T_A, Z^{u_i,z}(T_A))(\omega)\chi_{\{\tau_S \geq T_A\}}(\omega) &= \liminf_{i \to \infty} V(T_A, Z^{u_i,z}(T_A))(\omega) \\ &\geq V(T_A, z + \Delta Z(T_A)(\omega)) \\ &= V(T_A, z + \Delta Z(T_A))(\omega)\chi_{\{z \in S\}}(\omega). \end{aligned} \tag{141}$$

By other side, if $z \notin S$, it is trivially true that

$$\begin{aligned} \liminf_{i \to \infty} V(T_A, Z^{u_i,z}(T_A))(\omega)\chi_{\{\tau_S \geq T_A\}}(\omega) &\geq 0 \\ &= V(T_A, z + \Delta Z(T_A))(\omega)\chi_{\{z \in S\}}(\omega). \end{aligned} \tag{142}$$

because the value function is greater than zero.

Finally, applying the same steps as in the proof of Lemma L1' we have

$$\liminf_{s \to T_A -} V(s, z) \geq E\left[V(T_A, z + \Delta Z(T_A, z))\chi_{\{z \in S\}}\right]. \tag{143}$$

■

B Numerical Algorithm

In this appendix we describe the numerical algorithm in details for the case studied in section 4. The algorithm's properties are developed in Chancelier *et al.* (2007) and are described in Oksendal and Sulem (2007, Chapter 9) as well. First we describe the time invariant case (consistent with $t \geq T_A$) and then we incorporate the time variation.

B.1 Discrete Definitions

For $t \geq T_A$ we have the analytical solution but we provide the algorithm for this case and then discuss the difference for $t < T_A$. We shall solve the quasivariational inequality

$$\max\{A\Phi, g - \Phi\} = 0 \tag{144}$$

where the generator[21] A is

$$A\Phi = \frac{\partial \Phi}{\partial s} + \alpha x \frac{\partial \Phi}{\partial x} + \frac{1}{2}\beta x^2 \frac{\partial^2 \Phi}{\partial x^2}, \tag{145}$$

[21] Strictly speaking, the generator isn't a differential operator. Nonetheless it coincides in the set of twice differentiable functions with compact support. See theorem 1.22 in Oksendal and Sulem (2007).

and define the continuation region

$$\mathbf{C} = \left\{(s,x,\alpha) \in R^+ \times R^+ \times R^+ | \Phi(s,x) > g(s,x)\right\}. \tag{146}$$

Later we will define a grid but for now consider a "small" $h > 0$ and $h_t > 0$ and define a discrete version of A as

$$A_h v = \partial_t^{h_t} v + \alpha x \partial_x^h v + \frac{1}{2}\sigma^2 x^2 \partial_{xx}^{2,h} v, \tag{147}$$

where

$$\partial_t^{h_t} v(s,x) = \frac{v(s+h_t,x) - v(s,x)}{h_t}, \tag{148}$$

$$\partial_x^h v(s,x) = \frac{v(s,x+h) - v(s,x)}{h}, \tag{149}$$

$$\partial_{xx}^{2,h} v(x,y) = \frac{v(s,x+h) - 2v(s,x) + v(s,x-h)}{h^2}. \tag{150}$$

Let $T_h(s,\alpha)$ be the discrete version of a temporal slice of $\mathbf{C}$

$$T_h(s,\alpha) = \left\{ih | e^{-\rho s}\left(A_h\phi - \rho\phi\right) > e^{-\rho s}\widehat{g}(x) - e^{-\rho s}\phi\right\}.$$

where $e^{-\rho s}\widehat{g}(x) = g(s,x)$ and $\Phi(s,x) = e^{-\rho s}\phi(x)$.

B.1.1 Refinements for $t \geq T_A$

In our case, it is possible to make a transformation after T_A

$$\Phi(s,x) = e^{-\rho s}\phi(x) \tag{151}$$

and

$$\begin{aligned} A\Phi(s,x) &= \frac{\partial\left[e^{-\rho s}\phi(x)\right]}{\partial s} + e^{-\rho s}\alpha x \frac{\partial\phi(x)}{\partial x} + e^{-\rho s}\frac{1}{2}\beta x^2 \frac{\partial^2\phi(x)}{\partial x^2} \\ A\Phi(s,x) &= -\rho e^{-\rho s}\phi(x) + e^{-\rho s}\frac{\partial\left[\phi(x)\right]}{\partial s} + e^{-\rho s}\alpha x \frac{\partial\phi(x)}{\partial x} + e^{-\rho s}\frac{1}{2}\beta x^2 \frac{\partial^2\phi(x)}{\partial x^2}. \end{aligned} \tag{152}$$

$$A\Phi(s,x) = e^{-\rho s}A\phi - \rho e^{-\rho s}\phi(x) \tag{153}$$

Now we have an ordinary differential equation in x. In the region where $A\Phi(s,x) = 0$ we may rewrite

$$A\Phi(s,x) = 0 \tag{154}$$

if, and only if

$$A\phi - \rho\phi(x) = 0. \tag{155}$$

and in the discrete verstion

$$\begin{aligned} A_h\phi - \rho\phi(x) &= 0 && (156)\\ \alpha x \partial_x^h \phi(x) + \frac{1}{2}\sigma^2 x^2 \partial_{xx}^{2,h}\phi(x) - \rho\phi(x) &= 0. && (157) \end{aligned}$$

The computer can‘t handle an infinite number of elements. Then we will truncate the problem. Define the grid as $D_h = (ih)$ where $i \in \{0, ..., N\}$ and N are large enough to not compromise the results or to entail a small error. It is necessary to define a boundary condition at $x = Nh$. At the boundary of D_h we will consider the Neumann boundary condition

$$\frac{\partial \phi}{\partial x}(Nh) = 0. \qquad (158)$$

Fortunately this boundary condition is innocuous for the numerical results in section 4 because the continuation region is smaller then D_h. Remember that the region U is a sub-set of the continuation region and is defined as

$$U = \left\{(x, s, \alpha) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ | Ag + f > 0\right\} \qquad (159)$$

and we can define a discrete version (in our case $f = 0$) at time s

$$U_h(s, \alpha) = \{ih | A_h g(ih) - \rho g(ih) > 0\}. \qquad (160)$$

Note that for $t \geq T_A$ the set U_h above doesn't change with time. If possibe D_h shall be greater than U_h (this is indeed the case for the section 4).

Then the integrovariational inequality

$$\max\left\{e^{-\rho s}\left(A_h\phi - \rho\phi\right), g - e^{-\rho s}\phi\right\} = 0 \qquad (161)$$

may be written as

$$\begin{aligned} A_h\phi\,(ih) - \rho\phi\,(ih) &= 0 \text{ for } ih \in T_h, && (162)\\ e^{-\rho s}\phi &= g \text{ for } ih \notin T_h. && (163) \end{aligned}$$

and the slice of the continuation region is defined by

$$T_h(s, \alpha) = \left\{ih | e^{-\rho s}\left(A_h\phi - \rho\phi\right) > e^{-\rho s}\widehat{g}(x) - e^{-\rho s}\phi\right\} \qquad (164)$$

where $g(s, x) = e^{-\rho s}\widehat{g}(x)$. Note that T_h doesn't depend upon time after T_A.

B.2 The Algorithm

After defining the elements, the definition of the algorithm are now in order. Given the solution ϕ it is possible to find the continuation region $T_h(s,\alpha)$. On the other hand, given the continuation region, it is possible to find the solution ϕ. It seems a fixed point problem and one can guess if there is an iteration procedure leading to ϕ. Indeed Chancelier *et al.* (2007) shows that a slight different but equivalent problem has this feature. Instead of using the integrovariational inequality (161) one can use a better behaved and equivalent problem

$$\phi_h(x) = \max\left\{\left[I_h + \frac{\xi(A_h - \rho)}{1+\xi\rho}\right]\phi, \widehat{g}\right\} \tag{165}$$

where $0 < \xi \leq \min \frac{1}{|(A_h)_{ii}+\rho|}$, and I_δ is the identity operator ($I_h v_h = v_h$).

Again, this implies

$$A_h\phi\,(ih) - \rho\phi\,(ih) = 0 \text{ for } ih \in T_h, \tag{166}$$

$$e^{-\rho s}\phi = g \text{ for } ih \notin T_h. \tag{167}$$

but the slice of the continuation region is now defined as

$$T_h(s,\alpha) = \left\{ih| \left[I_h + \frac{\xi(A_h - \rho)}{1+\xi\rho}\right]\phi(ih) > \widehat{g}\right\}. \tag{168}$$

This difference allows us to define an iteration procedure converging to the right solution:

- (step n, sub-step 1) Given v^n find T_h^{n+1} such that

$$T_h^{n+1}(s,\alpha) = \left\{ih| \left[I_h + \frac{\xi(A_h - \rho)}{1+\xi\rho}\right]\phi(ih) > \widehat{g}\right\}. \tag{169}$$

- (step n, sub-step 2) Compute v^{n+1} as the solution of

$$A_h v^{n+1}\,(ih) - \rho v^{n+1}\,(ih) = 0 \text{ for } ih \in T_h^{n+1}, \tag{170}$$

$$e^{-\rho s}v^{n+1} = g \text{ for } ih \notin T_h^{n+1}. \tag{171}$$

- Repeat the procedure until $\max\left\{abs(v^{n+1} - v^n)\right\}$ less then a predefined error.

The only piece missing is to define v^0 or T_h^0. In this case, it is easier to define $T_h^0 = D_h$ and begin the procedure from sub-step 2. It is shown that $\lim_{n\to\infty} v^n \to \phi$.

Remark 17 *We omit several technical conditions in the above presentation. They hold for the problem we are dealing with and we refer to Oksendal and Sulem (2007) and Chancelier et al. (2007) in order to account for them.*

B.3 Modification in the algorithm for $t < T_A$

We will discretize the time and apply the above algorithm at each slice of time using a implicit scheme. Note that it is necessary to define a boundary condition at $t = T_A$. Remember that we have the analytical solution after T_A. We have for $t = T_A$ the boundary condition

$$\widetilde{\Phi}(T_A, x) = E\left[\Phi\left(T_A, x, \alpha\right)\right], \tag{172}$$

$$\widetilde{\Phi}(T_A, x) = E\left[e^{-\rho s} C(\alpha) x^{\lambda_1(\alpha)} \chi_{\{0<x<x^*(\alpha)\}} + e^{-\rho s}(x-a)\chi_{\{x^*(\alpha)\leq x\}}\right] \tag{173}$$

$$\widetilde{\Phi}(T_A, x) = \int_{\underline{\alpha}}^{\overline{\alpha}} \left(e^{-\rho s} C(\alpha) x^{\lambda_1(\alpha)} \chi_{\{0<x<x^*(\alpha)\}} + e^{-\rho s}(x-a)\chi_{\{x^*(\alpha)\leq x\}}\right) d\alpha \tag{174}$$

where $C(\alpha)$, $\lambda_1(\alpha)$ and $x^*(\alpha)$ are defined in section 4.

Note that Φ depends upon α and it changes after T_A. Nonetheless, before it doesn't change. Fot $t < T_A$ we omit α in the notation

$$\begin{aligned} \Phi\left(T_A - nh_t, ih\right) &= \Phi\left(T_A - nh_t, ih, \alpha\left(T_A - nh_t\right)\right) \\ &= \Phi\left(T_A - nh_t, ih, \alpha\left(0\right)\right). \end{aligned} \tag{175}$$

The grid in the dimension x will be the same for all s and the discretization in time will be given by $T_A - nh_t$. Now the continuation region varies over time, $T_h(s) = T_h(T_A - nh_t)$, and we have

$$A_h \Phi\left(T_A - nh_t, ih\right) = 0 \qquad \text{for } ih \in T_h(T_A - nh_t), \tag{176}$$

$$\Phi\left(T_A - nh_t, ih\right) = g\left(T_A - nh_t, ih\right) \quad \text{for } ih \notin T_h\left(T_A - nh_t\right), \tag{177}$$

with Neumann boundary condition at $x = Nh$

$$\partial_x^h v(s, Nh) = 0 \tag{178}$$

and the final condition

$$\Phi\left(T_A, ih\right) = \widetilde{\Phi}(T_A, x). \tag{179}$$

Note that we defined the discrete time differential as

$$\partial_t^{h_t} v(s,x) = \frac{v(s+h_t, x) - v(s,x)}{h_t}. \tag{180}$$

This entails a implicit scheme when solving the numerical partial differential equation defined in equations (176) and (177). For instance, given $T_h^0(T_A - h_t)$,we have for $s = T_A - h_t$

$$\begin{aligned} A_h \Phi(T_A - h_t, ih) &= \partial_t^{h_t}\Phi + \alpha \partial_x^h \Phi + \frac{1}{2}\sigma^2 x^2 \partial_{xx}^{2,h}\Phi \\ &= \frac{\widetilde{\Phi}(T_A, ih) - \Phi(T_A - h_t, ih)}{h_t} + \alpha x \partial_x^h \Phi(T_A - h_t, ih) + \frac{1}{2}\sigma^2 x^2 \partial_{xx}^{2,h}\Phi(T_A - h_t, ih) \end{aligned}$$

and

$$\begin{aligned} A_h\Phi(T_A - h_t, ih) &= 0 && \text{for } ih \in T_h(T_A - nh_t). \\ \Phi\left(T_A - nh_t, ih\right) &= g\left(T_A - nh_t, ih\right) && \text{for } ih \notin T_h\left(T_A - nh_t\right), \end{aligned} \tag{181}$$

with the Neumann boundary conditions. Now it is only necessary to use the algorithm defined above in this slice of time.

The problem may be solved sequentially as $\Phi(T_A - nh_t, ih)$ depends upon $\widetilde{\Phi}$ only through $\Phi(T_A - (n-1)h_t, ih)$. Moreover,$\Phi(T_A - (n-1)h_t, ih)$ doesn't depend upon $\Phi(T_A - nh_t, ih)$.

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