On the combination of multivariate volatility forecasts

Abstract

We devise a novel approach to combine predictions of conditional covariance matrices using economic criteria based on portfolio selection. The combination scheme takes into account not only the portfolio objective function but also the portfolio characteristics in order to define the mixing weights. The proposed model combination i) does not require a proxy for the latent conditional covariance matrix, ii) does not require optimization of the combination weights, which facilitates its implementation in practical situation, and iii) holds the equally weighted model combination as a particular case. An empirical application involving 4 data sets with cross section dimensions ranging from 17 to 100 assets over a 10-year time span shows that combined multivariate GARCH forecasts leads to minimum variance portfolios with lower risk on an out-of-sample basis with respect to all individual models considered. The results are robust to the portfolio re-balancing frequency.
Key words: Bootstrap, conditional correlation models, mean-variance portfolios, Sharpe ratios, transaction costs.

1 Introduction

Combining predictions from alternative models is a very well established forecasting approach. In fact, the literature on forecast combinations is extensive and points to the superiority of combined forecasts with respect to single models in many different contexts. The motivation to combine forecasts comes from an important result from the methodological literature on forecasting, which shows that a linear combination of two or more forecasts may yield more accurate predictions than using only a single forecast (Granger, 1989; Newbold and Harvey, 2002; Aiolfi and Timmermann, 2006). Moreover, adaptive strategies for combining forecasts might also mitigate structural breaks and model misspecification and thus lead to more accurate forecasts (Pesaran and Timmermann, 2007).

A number of potential explanations for the good performance of combined forecasts vis-a-vis individual forecast models can be pointed out. First, if two models provide partial, but incompletely overlapping explanations, then some combination of the two might do better than either alone; see Hendry and Clements (2004). Specifically, if two forecasts were differentially biased (one upwards, one downwards), then combining could be an improvement over either. Second, averaging forecasts reduces variance to the extent that separate sources of information are used. Third, forecast combination can also alleviate the

problem of model uncertainty.[1]

Existing evidence suggests that the combination of univariate volatility leads to more accurate forecasts with respect to single models (Becker and Clements, 2008; Patton and Sheppard, 2009). The study of combined multivariate volatility predictions, however, is in its early days. This issue is very important since the combination of multivariate volatility models might also lead to more accurate predictions, therefore improving decision making in economic and financial problems that depend on covariance estimates such as asset pricing, portfolio optimization and market risk management. Engle and Colacito (2006), for instance, show that accurate covariance information will allow the investor to achieve lower volatility, higher return, or both.

In this paper, we put forward a novel approach to combine multivariate volatility predictions from alternative conditional covariance models. The proposed combination rule is motivated by the fact this class of models is often applied in portfolio selection problems. More specifically, combination weights are defined by taking into account the past performance of each individual model in obtaining optimal portfolios. The past performance can be either in-sample or out-of-sample. In our empirical implementation of the proposed combination method, we rely on in-sample rather than out-of-sample portfolio performance of each individual model. This choice is motivated by two reasons. First, it simplifies implementation as the same estimation window used to estimate the parameters of each individual model is also used to evaluate portfolio performance. Second, and most importantly, we performed extensive robustness checks to evaluate the gains (or lack of) from changing to past out-of-sample portfolio performance, despite higher computational cost. The results are very similar in comparison to those based on the past in-sample performance of each individual model, which makes us confident to employ this approach. Another important aspect of the proposed approach is that combination weights can be calibrated in order to adjust i) the aggressiveness in the allocation across alternative models (i.e. a cross section adjustment) and ii) the importance given to the most recent observations in the calculation of portfolio performance (that is, a time series adjustment). Three major advantages of this approach are that i) does not require a proxy for the latent conditional covariance matrix, ii) does not require optimization of the combination weights, and iii) holds the equally-weighted model combination as a particular case. Each of these advantages are discussed in Section 2.

The proposed approach to combine predictions of conditional covariance matrices is based on the mean-

[1] Geweke and Amisano (2011) study the existence of model complementarities in linear prediction pools and show that a model with positive weight in a pool may have zero weight if some other models are deleted from that pool.

variance portfolio selection problem introduced by Markowitz (1952). Two aspects motivate our choice for adopting the mean-variance framework. First, economic applications involving multivariate volatility models often rely on the mean-variance portfolio problem; see, for example, Engle and Colacito (2006) and Engle and Sheppard (2008). Therefore, it is natural to consider this portfolio selection policy when devising a model combination approach for conditional covariance models. Second, the mean-variance framework is one of the milestones of modern finance theory and is widespread among academics and market participants. In this framework, individuals choose their allocations in risky assets based on the trade-off between expected return and risk. We consider two alternative versions of the mean-variance portfolio problem. The first is based on an investor who wishes to minimize portfolio risk subjected to a target portfolio return. In this case, the investor wishes to achieve higher risk-adjusted portfolio returns. The mean-variance problem, however, is known to be very sensitive to estimation of the mean returns (e.g. Jagannathan and Ma, 2003). Very often, the estimation error in the mean returns degrade the overall portfolio performance and introduces an undesirable level of portfolio turnover. In fact, existing evidence suggest that the performance of optimal portfolios that do not rely on estimated mean returns is better; see, for instance, DeMiguel *et al.* (2009). Because of that, we also consider a second type of investor who adopts the minimum variance criterion in order to decide her portfolio allocations. This portfolio policy can be seen as a particular case of the traditional mean-variance optimization. In fact, existing evidence suggest that the performance of optimal portfolios that do not rely on estimated mean returns is better; see, for instance, DeMiguel *et al.* (2009).

It is worth noting that both mean-variance and minimum variance portfolio problems require estimation of the full covariance matrix of asset returns. In most practical situations, however, the investor faces uncertainty on which is the most appropriate specification to model and to forecast covariances. For instance, should the investor employ a multivariate GARCH model or instead a multivariate stochastic variance (SV) specification? Should the investor consider time varying or time invariant conditional correlations? Are more parameterized specifications better than more parsimonious ones? How to handle this uncertainty? All these questions are typically answered in the literature by horse-racing many single models; see, for instance, Engle and Colacito (2006) and Engle and Sheppard (2008). Our combination approach helps alleviating this problem since it is explicitly built to exploit model complementarities and to favor the models that generate portfolios with better portfolio performance and to penalize those that yield portfolios with poor performance.

Our paper is related to recent studies such as Becker *et al.* (2014) and Amendola and Storti (2015). Becker *et al.* (2014) study the ability of alternative loss functions to select the best specifications for the portfolio selection problem and find that a likelihood-based function is the best function for this purpose. Amendola and Storti (2015) pioneered the literature on combined multivariate volatility predictions and extended the approach in Patton and Sheppard (2009) to a multivariate setting. Their approach involves estimating model combination weights via minimization of a loss function and similar as in Becker *et al.* (2014) requires a proxy for the unobserved latent covariance matrix, which is specified as a realized covariance measure. Our approach, in contrast, differs is several aspects since does not require neither optimization of combination weights nor a proxy for the unobserved covariance matrix, which greatly facilitates its implementation in practical situations. Moreover, instead of adopting a pure statistical criterion for estimating the mixing weights, we directly apply an economic criterion for defining, in a dynamic fashion, how much weight to put in each model. Therefore, our approach to combine multivariate volatility predictions is consistent with the fact that this class of models is ultimately applied to economic problems.

We test the effectiveness of the proposed model combination using 4 data sets from the US, European and Asian stock markets with cross section dimensions ranging from 17 to 100 assets over a 10-year time span. We implement a set of 8 alternative multivariate GARCH models that are widely used in portfolio selection problems and obtain model combination schemes using the mean-variance and minimum variance portfolio criterion for the mixing. We conduct a detailed economic evaluation of the resulting optimal portfolios and implement a test for the portfolio risk and for the portfolio Sharpe ratio based on the bootstrap procedure of Politis and Romano (1994), which allows us to formally compare optimal portfolios obtained with alternative specifications in terms of their sample characteristics.

Our results can be summarized as follows. First, when combining multivariate volatility forecasts based on the mean-variance criterion, we observe that the resulting conditional covariance estimators yield mean-variance portfolios with higher risk-adjusted returns measured by the Sharpe ratio. Second, when combining multivariate volatility forecasts based on the minimum variance criterion, we observe that the minimum variance portfolios obtained with the combined estimators are substantially and statistically less risky with respect to all individual models considered.

Our baseline results assume that optimal portfolios are re-balanced in a daily frequency. The transaction costs involved in this re-balancing frequency, however, might degrade the performance of the port-

folios and hinder its implementation in practice. In this sense, we also assess the impacts of transaction costs by implementing the mean-variance and minimum variance portfolios under weekly and monthly re-balancing frequencies. We find that, as expected, lowering the portfolio re-balancing frequency leads to a substantial decrease in portfolio turnover. However, we observe that lowering the re-balancing frequency worsen portfolio performance in most of the cases, for both individual models and their combinations. However, combined forecasts still outperform individual models in when portfolios are re-balanced in a lower frequency.

The rest of the paper is organized as follows. In section 2 we detail the proposed combination rule. Section 3 provide an empirical application based on two alternative data sets with many assets. Finally, section 4 concludes.

2 Basic idea

In this section, we detail the proposed approach to combine multivariate volatility predictions in a dynamic fashion. To solve a variety of important problem in finance, it is common to choose a conditional covariance specification in order to account for the time variation in second order moments; see, for instance, Engle and Colacito (2006) and Becker *et al.* (2014). In most practical situations, however, the volatility process is subject to changes and the investor faces uncertainty on which is the most appropriate specification to model and to forecast covariances at a given point in time. In order to identify the best specification, the most common practice in the literature is to horse-race many single models. We, on the other hand, consider the possibility of improving the portfolio performance via model combinations. More specifically, we consider the case in which there exists M alternative candidates. In this case, the combined conditional covariance estimator, H_t^{Comb}, is defined as

$$H_t^{Comb} = \lambda_{1,t} H_t^1 + \ldots + \lambda_{M,t} H_t^M, \tag{1}$$

where H_t^m denotes conditional covariance matrix of the m-th candidate model and $\lambda_{m,t}$ is the corresponding combination weight such that $\sum_{m=1}^{M} \lambda_{m,t} = 1$ and $\lambda_{m,t} \geq 0 \, \forall \, m$. It is worth noting that a linear combination in (4) is conveniently chosen as it guarantees that the resulting H_t^{Comb} is positive-definite provided that individual models also deliver positive-definite conditional covariance matrices.

The most important aspect when combining alternative model based forecasts is to specify a combi-

nation vector $\lambda_t = \{\lambda_{1,t}, \ldots, \lambda_{M,t}\}$, that is, to decide how much weight to put in each individual forecast. The literature on forecast combinations offers many options to specify λ_m. In the case of volatility models, for instance, Patton and Sheppard (2009) and Amendola and Storti (2015) consider a combination scheme based on the minimization of a statistical loss function that requires a proxy for the true unobserved volatility, which is specified as a realized volatility measure. Despite the fact that the realized covariance matrix is only an estimate of the true volatility, higher frequency data to compute this measure is not always available and the combination weights do not incorporate any information about the decision making task in which forecasts will be ultimately used.

We take a different approach with respect to the existing literature and specify λ_m in (4) by taking into account an economic criterion based on the mean-variance portfolio optimization problem. Using this economic criterion has at least two appealing features. First, it is consistent with the fact that multivariate volatility models are ultimately applied to economic problems. Second, it is based on a very well established portfolio selection problem. Next we detail the economic gain functions used to construct the combined conditional covariance estimator in (4).

2.1 Mean-variance model combination

We consider an investor who allocates her wealth in N alternative risky assets. In order to choose the weights w_i for $i = 1, \ldots, N$ of each asset in the portfolio, we assume the investor adopts the mean-variance portfolio policy. In this setting, the investor wishes to minimize portfolio risk subjected to a target portfolio return. This portfolio optimization problem is defined as

$$\begin{aligned} &\min_{w \in \Re^N} \; w_t' H_t w_t \\ &\text{subject to} \\ &w_t' \mu = \mu_0 \\ &\sum_{i=1}^{N} w_i = 1, \end{aligned} \tag{2}$$

where w_t is the vector of portfolio weights for time t chosen at time $t-1$ and H_t is a positive-definite conditional covariance matrix of asset returns for time t forecasted at time $t-1$. μ is the vector of expected returns and μ_0 is the required return. In our empirical implementation, we define μ_0 as the return of the equally-weighted portfolio. The solution to (2) is usually obtained numerically. Upon solving the portfolio problem in (2) for a sample with T observations, the investor is interested in computing the

risk-adjusted returns, which is usually measured by the Sharpe ratio. the investor computes the sample portfolio variance as

In this case, the combined conditional covariance estimator in (4) can privilege the models that yield higher risk-adjusted returns and penalize those that yield lower risk-adjusted returns. A common measure of risk-adjusted returns is the sample Sharpe ratio (SR) defined as $SR = \hat{\mu}/\hat{\sigma}$ where $\hat{\mu}$ is the average sample portfolio return. In this case, the combination vector λ_m can be defined as

$$\lambda_m = \frac{\left(\hat{\mu}_m/\hat{\sigma}_m\right)^{\eta}}{\sum\limits_{m=1}^{M}\left(\hat{\mu}_m/\hat{\sigma}_m\right)^{\eta}},$$

where $\hat{\mu}_m$ is the average sample portfolio return obtained with the m-th candidate model. In order to rule out the possibility of selecting models that generate portfolios with negative SR, one can set $\lambda_m = 0$ if $SR_m < 0$. We refer to this approach as the mean-var(η) combination rule. Therefore, the resulting combined conditional covariance estimator can be defined as

$$H_t^{Comb} = \frac{1}{\sum\limits_{m=1}^{M}\left(\hat{\mu}_m/\hat{\sigma}_m\right)^{\eta}} \sum_{m=1}^{M}\left(\hat{\mu}_m/\hat{\sigma}_m\right)^{\eta} H_t^m.$$

2.2 Minimum variance model combination

we assume the investor adopts the minimum variance portfolio policy. This portfolio policy can be seen as a particular case of the traditional mean-variance optimization. The mean-variance problem, however, is known to be very sensitive to estimation of the mean returns (e.g. Jagannathan and Ma, 2003). Very often, the estimation error in the mean returns degrade the overall portfolio performance and introduces an undesirable level of portfolio turnover. In fact, existing evidence suggest that the performance of optimal portfolios that do not rely on estimated mean returns is better; see, for instance, DeMiguel *et al.* (2009).

The minimum variance portfolio problem is defined as

$$\begin{aligned} &\min_{w\in\Re^N} \; w_t' H_t w_t \\ &\text{subject to} \\ &\sum_{i=1}^{N} w_i = 1, \end{aligned} \tag{3}$$

where w_t is the vector of portfolio weights for time t chosen at time $t-1$ and H_t is a positive-definite conditional covariance matrix of asset returns for time t forecasted at time $t-1$. The solution to (3) is defined as $w_t = \iota' H_t^{-1} / \iota' H_t^{-1} \iota$, where ι is a vector of ones. Upon solving the portfolio problem in (3) for a sample with T observations, the investor computes the sample portfolio variance as $\hat{\sigma}^2 = \frac{\sum_{t=1}^{T-1} (w_t{}' R_{t+1} - R_p)^2}{T-1}$, where R_{t+1} is the vector of asset returns at time $t+1$ and R_p is the average portfolio return.

When solving the portfolio problem in (3), it is common to choose a conditional covariance specification for H_t in order to account for the time variation in second order moments; see, for instance, Engle and Colacito (2006) and Becker *et al.* (2014). In most practical situations, however, the investor faces uncertainty on which is the most appropriate specification to model and to forecast covariances. In order to identify the specification that minimize the sample portfolio variance, $\hat{\sigma}^2$, the most common practice in the literature is to horse-race many single models. We, on the other hand, consider the possibility of improving the portfolio performance via model combinations. More specifically, we consider the case in which there exists M alternative candidates. In this case, the combined conditional covariance estimator, H_t^{Comb}, is defined as

$$H_t^{Comb} = \lambda_1 H_t^1 + \ldots + \lambda_M H_t^M, \tag{4}$$

where H_t^m denotes conditional covariance matrix of the m-th candidate model and $\lambda_{m,t}$ is the corresponding combination weight such that $\sum_{m=1}^{M} \lambda_m = 1$ and $\lambda_m \geq 0$. It is worth noting that a linear combination in (4) is conveniently chosen as it guarantees that the resulting H_t^{Comb} is positive-definite provided that individual models are also positive-definite.

The most important aspect when combining alternative model based forecasts is to specify a combination vector λ_m in (4), that is, to decide how much weight to put in each individual forecast. The literature on forecast combinations offers many options to specify λ_m. In the case of volatility models, for instance, Patton and Sheppard (2009) and Amendola and Storti (2015) consider a combination scheme based on the minimization of a loss function that requires a proxy for the true unobserved volatility, which is specified as a realized volatility measure.

We take a different road with respect to the existing literature and specify λ_m in (4) by taking into account an economic criterion based on the portfolio optimization problem in (3). The criterion privileges the models that generate portfolios with lower variance and penalizes those that yield portfolios with

higher variances. This definition boils down to a combination rule defined as

$$\lambda_m = \frac{\left(1/\hat{\sigma}_m^2\right)}{\sum_{m=1}^{M}\left(1/\hat{\sigma}_m^2\right)}, \qquad m = 1, \ldots, M \tag{5}$$

where $\hat{\sigma}_m^2$ is the sample variance of the minimum variance portfolios obtained with the m-th candidate model based on a sample of T observations. Instead of computing $\hat{\sigma}_m^2$ as the sample variance, we follow Stock and Watson (2004) and Genre *et al.* (2013) and consider a more general expression for the model's performance metric by introducing a discount (or forgetting) factor δ, i.e.

$$\hat{\sigma}_m^2 = \frac{\sum_{t=1}^{T-1} \delta^{T-1-t} \left(w_t{}' R_{t+1} - R_p\right)^2}{T-1}. \tag{6}$$

Values of δ which are below unity assign higher weights to more recent portfolio variances in the calculation of combination weights. In our implementation, we follow Genre *et al.* (2013) and set $\delta = \{1, 0.95, 0.85\}$. The case where $\delta = 1$ corresponds to no discounting and is equivalent to the common expression of the sample variance. We refer to the combination rule in (5) and (6) as the min-var(δ) combination rule. A portfolio policy that takes into account the inverse of the sample variance of the individual assets has been also proposed in Kirby and Ostdiek (2012). The authors refers to this policy as volatility timing strategies.

It is also worth noting that the min-var(δ) strategy in (5) and (6) belongs to a more general class of min-var combination with weights of the form

$$\lambda_m = \frac{\left(1/\hat{\sigma}_m^2\right)^\eta}{\sum_{m=1}^{M}\left(1/\hat{\sigma}_m^2\right)^\eta}, \qquad m = 1, \ldots, M. \tag{7}$$

The idea behind this generalization is straightforward. The tuning parameter $\eta \geq 0$ determines how aggressively we adjust the mixing weights in response to changes in the realized portfolio variance obtained with each of the candidate models. As $\eta \to 0$ we recover the equally weighted model combination, and as $\eta \to \infty$ the weight on the model that yields the lowest realized portfolio variance approaches 1. In our implementation, we follow Kirby and Ostdiek (2012) and consider alternative values of η such that $\eta = \{1, 2, 4, 10\}$. We refer to the mixing in (7) as the min-var(δ, η) combination rule. Finally, substituting

equation (7) into (4) yields a more general expression for the min-var(δ, η) combined estimator:

$$H_t^{Comb} = \frac{1}{\sum_{m=1}^{M} \left({}^{1}\!/_{\hat{\sigma}_m^2} \right)^{\eta}} \sum_{m=1}^{M} \left[\left({}^{1}\!/_{\hat{\sigma}_m^2} \right)^{\eta} H_t^m \right]. \tag{8}$$

The combined conditional covariance estimator in (8) uses the past performance of each individual model in delivering less risky minimum variance portfolios in order to define the combination weights.[2] The aggressiveness in the allocation across alternative models is calibrated by the tuning parameter η. Moreover, the importance given to the most recent observations in the calculation of portfolio variances is calibrated by the discount factor δ. Therefore, while the η parameter performs a cross section adjustment on the aggressiveness in the allocation across alternative models, the discount factor δ performs a time series adjustment as it controls for the importance of most recent observations.

It is also worth emphasizing four important aspects of the model combination in (8). First, the mixing does not require a proxy for the latent conditional covariance matrix. This proxy is usually defined by means of a realized covariance matrix based on high-frequency data as in Laurent *et al.* (2012), Laurent *et al.* (2013), and Becker *et al.* (2014). The proposed approach, on the other hand, dispenses the use of intraday data and can be implemented with data sampled at any frequency. Second, the mixing rule does not require optimization of the combination weights, which facilitates its implementation in practice.

Finally, the fourth important aspect of the of the proposed approach to combine multivariate volatility predictions is that it holds the equally weighted model combination as a particular case (when $\eta = 0$). This specific model combination is found to outperform more sophisticated combination schemes in many contexts; see, for instance, Clemen (1989), De Menezes *et al.* (2000), Wallis (2011), and Genre *et al.* (2013). Some authors have argued this result is due to the instability of combination weights, which can deteriorate the performance of optimal combinations; see Kang (1986). This instability has its roots in the sampling error, which contaminates the estimated weights and is exacerbated by the collinearity that typically exists among primary forecasts (Diebold, 1989). The imposition of equal weights eliminates variation in the estimated weights and increases robustness with respect to model uncertainty, time-variation of parameters, and estimation errors that arise when combination weights have to be estimated (Palm and Zellner, 1992). Finally, Armstrong (2001) and Timmermann (2006) establish conditions under which adopting the equally weighted model combination is optimal. Timmermann (2006), for instance, argues

[2] Alternative algorithms to combine predictions for the conditional mean based on the past performance of the candidate models are also proposed in Yang (2004) and Hibon and Evgeniou (2005).

that equal weights are optimal in situations with an arbitrary number of forecasts when the individual forecast errors have the same variance and identical pairwise correlations. These arguments motivate us to choose the equally weighted model combination min-var($\eta = 0$) as the main benchmark among all individual and combined covariance specifications considered in the paper.

2.3 Modeling H_t

We assume that the multivariate system of asset returns is modeled as $R_t = z_t H_t^{1/2}$. To model H_t, it is possible to consider a broad set of conditional covariance specifications including multivariate GARCH and SV models; see Asai *et al.* (2006), Bauwens *et al.* (2006), Silvennoinen and Teräsvirta (2009), and Chib *et al.* (2009). In this paper, we follow Engle and Sheppard (2008) and Becker *et al.* (2014) and restrict our attention to the former class and implement a set of $M = 8$ alternative multivariate GARCH specifications widely used in portfolio selection problems:

Exponentially weighted moving average (EWMA): The EWMA model is defined as

$$H_t = \alpha R_{t-1}{}' R_{t-1} + (1 - \alpha) H_{t-1},$$

where α is a nonnegative parameter. When the α is set to a fixed value of 0.04, the EWMA is equivalent to the popular Riskmetrics^{TM} approach. Zaffaroni (2008) shows that although it permits sizable computational gains and provide a simple way to impose positive semi-definitiveness of the resulting conditional covariance matrices, the Riskmetrics^{TM} delivers non-consistent parameter estimates. Therefore, in our implementation of the EWMA specification the parameter α is estimated via maximum likelihood; see details below.

Optimal rolling estimator (ORE): The general rolling estimator is defined as $H_t = \sum_{t=1}^{\infty} \Omega_{t-k} \odot R_{t-k}{}' R_{t-k}$, where Ω_{t-k} is a symmetric matrix of weights and $\odot$ denotes the element-by-element multiplication. The optimal weighting scheme proposed in of Foster and Nelson (1996) is given by $\Omega_{t-k} = \alpha \exp(-\alpha k) \iota' \iota$ where ι is a vector of ones. Therefore, the general rolling estimator can be rewritten as

$$H_t = \alpha \exp(-\alpha) R_{t-1}{}' R_{t-1} + \exp(-\alpha) H_{t-1},$$

where α is a nonnegative parameter. The ORE specification has been applied in many portfolio selection problems such as in Fleming *et al.* (2001, 2003) and Pooter *et al.* (2008). Fleming *et al.* (2003), in particular, point out that covariance matrix forecasts based on the ORE specification results in better portfolios in comparison to those obtained with other (unrestricted) multivariate GARCH models. The authors argue that the smoothness of the rolling estimator as the main reason for this.

Scalar VECH: The scalar VECH specification of Bollerslev *et al.* (1988) is defined as

$$H_t = C'C + \alpha R_{t-1} R_{t-1}{}' + \beta H_{t-1}.$$

Instead of estimating $N(N+1)/2$ unique elements of C, we employ the variance targeting technique as suggested in Engle and Mezrich (1996). The general idea is to estimate the intercept matrix by an auxiliary estimator that is given by $\hat{C}'\hat{C} = \bar{S}\,(1-\alpha-\beta)$, where $\bar{S} = \frac{1}{T}\sum_{t=1}^{T} R_t R_t{}'$, thus yielding the variance-targeting scalar VECH model

$$H_t = \bar{S}\,(1-\alpha-\beta) + \alpha R_{t-1} R_{t-1}{}' + \beta H_{t-1},$$

which is covariance-stationary provided that $\alpha + \beta < 1$.

Orthogonal GARCH (O-GARCH): The O-GARCH model of Alexander (2001) belongs to a class of factor models and is able to achieve significant computational gains via dimensionality reduction. The O-GARCH model is given by $\Sigma_t = W\Omega_t W$, where W is a $N \times k$ matrix whose columns are given by the first k eigenvectors of the $t \times N$ matrix of asset returns, and Ω_t is a $k \times k$ diagonal matrix whose elements are given by $h_{f_{kt}}$ where $h_{f_{kt}}$ is the conditional variance of the k-th principal component and follows a GARCH(1,1) process. We implement the O-GARCH model using 3 principal components.

Conditional correlation models: This class of models is defined as $H_t = D_t \Psi_t D_t$, where D_t is a $N \times N$ diagonal matrix with diagonal elements given by $h_{i,t}$, where $h_{i,t}$ is the conditional variance of the i-th asset and follows a GARCH(1,1) process, and Ψ_t is a symmetric conditional correlation matrix with elements $\rho_{ij,t}$, where $\rho_{ii,t} = 1$, $i, j = 1, \ldots, N$. We consider 4 alternative specifications to model Ψ_t : (i) the constant conditional correlation (CCC) model of Bollerslev (1990), (ii) the dynamic conditional

correlation (DCC) model of Engle (2002), (iii) the asymmetric DCC (ASYDCC) of Cappiello *et al.* (2006), (iv) the dynamic equicorrelation (DECO) model of Engle and Kelly (2012). Engle and Colacito (2006) and Engle and Sheppard (2008) study the performance of alternative conditional correlation models in portfolio selection problems.

Multivariate GARCH models are typically estimated via quasi maximum likelihood (QML). However, this estimator is found to be severely biased in large dimensions; see, for instance, Engle *et al.* (2008) and Hafner and Reznikova (2012). In this paper, the parameters of the EWMA, ORE, and VECH specifications are estimated with the composite likelihood (CL) method proposed by Engle *et al.* (2008). As for the conditional correlation models, their estimation can be conveniently divided into volatility part and correlation part. The volatility part refers to estimating the univariate conditional variances which is done by QML assuming Gaussian innovations. The parameters of the correlation matrix in the DCC and ASYDCC models are estimated using the CL method. As pointed out by Engle *et al.* (2008), the CL estimator provides more accurate parameter estimates in comparison to the two-step procedure proposed by Engle (2002), especially in large problems.

Finally, it is important to emphasize that other multivariate GARCH specifications, apart from those considered in this paper, have been proposed in the literature; see Bauwens *et al.* (2006) and Silvennoinen and Teräsvirta (2009) for reviews. In this sense, the model set considered in this paper can be increased in many alternative ways.[3] Even though we consider these alternative specifications to be very interesting, we focus on a model set that includes 8 of the most widely used specifications. The computational effort to perform rolling window estimations of each of these 8 specifications for the high dimensional data sets considered in this paper is already immense, which limits our capacity to increase the model set.

[3]Important contributions in the literature on multivariate volatility models include the regime switching DCC of Pelletier (2006), the Wishart stochastic volatility model of Philipov and Glickman (2006a,b), the semiparametric model for the correlation dynamics of Hafner *et al.* (2006), the generalized DCC model of Billio and Caporin (2009) and Hafner and Franses (2009), the factor-DCC model of Zhang and Chan (2009), the DCC-MIDAS of Colacito *et al.* (2011), the HEAVY models of Noureldin *et al.* (2012), the factor-spline-GARCH model of Rangel and Engle (2012), the heterogeneous ASYDCC model of Asai (2013), the multivariate rotated ARCH model of Noureldin *et al.* (2014), the realized beta GARCH model of Hansen *et al.* (2014), the dynamic factor multivariate GARCH model of Santos and Moura (2014), the smooth transition conditional correlation model of Silvennoinen and Teräsvirta (2015), among many others.

3 Empirical application

3.1 Data sets

We conduct an empirical investigation on the performance of minimum variance portfolios obtained with individual models as well as with the min-var(δ, η) model combination. The investigation in based on daily observations of 4 alternative data sets from the global stock markets from 01/01/2004 until 31/12/2013. The first data set consists of closing prices of the 50 mostly traded stocks belonging to the US stock market index SandP100 (50SandP). The second data set consists of 100 US industry portfolios (100Ind) obtained from Ken French's web site. Both data sets have $T = 2517$ observations. The third data set consists of closing prices of 45 stocks belonging to the European stock market index Eurostoxx (45Euro). Finally, the fourth data sets consists of closing prices of 17 stocks belonging to the Asian stock market index STI (17STI). These last two data sets have, respectively, $T = 2504$ and $T = 2520$ observations.

3.2 Implementation details

Departing from the first $t = 1500$ observations, all models described in section 2.3 are estimated and their corresponding one-step-ahead forecasts of the conditional covariance matrix are obtained. We obtain minimum variance portfolio weights for each of the models along the t observations and compute the one-step-ahead forecast of the conditional combined conditional covariance estimator using the min-var(δ, η) combination rule in (8) for $\delta = \{1, 0.95, 0.85\}$ and $\eta = \{0, 1, 2, 4, 10\}$. When $\eta = 0$ the min-var(δ, η) combination rule corresponds to the equally weighted model combination irrespective of the value of δ. Finally, we add one observation to the estimation window and repeat the process until the end of the data set is reached. We end up with a sample of $T - t$ out-of-sample observations.

We use the out-of-sample observations to evaluate the minimum variance portfolio performance in terms of average return ($\hat{\mu}$), standard deviation (volatility) of returns ($\widehat{\sigma}$), Sharpe Ratio (SR), and

turnover. These statistics are calculated as follows:

$$\hat{\mu} = \frac{1}{T-1}\sum_{t=1}^{T-1} w_t' R_{t+1}$$

$$\hat{\sigma} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T-1} (w_t' R_{t+1} - R_p)^2}$$

$$SR = \frac{\hat{\mu}}{\hat{\sigma}}$$

$$\text{Turnover} = \frac{1}{T-1}\sum_{t=1}^{T-1}\sum_{j=1}^{N} (|w_{j,t+1} - w_{j,t}|),$$

where $w_{j,t}$ is the weight of the asset j in the portfolio in period t before the re-balancing, while $w_{j,t+1}$ is the desired weight of the asset j in period $t+1$. The turnover as defined above can be interpreted as the average fraction of wealth traded in each period.

In order to assess the relative performance of the model combination approach proposed in the paper, we consider as the benchmark covariance specification the min-var($\eta = 0$) combination, which corresponds to the equally weighted model combination. The stationary bootstrap of Politis and Romano (1994) with B=1000 resamples and block size $b = 5$ was used to test the statistical significance of differences between the standard deviation of minimum variance portfolio returns obtained with each individual model and with each model combination with respect to those obtained with the benchmark.[4] The methodology suggested in Ledoit and Wolf (2008) was used to obtain p-values.

3.3 Results

Table 1 reports performance statistics for the minimum variance portfolios obtained with each of the individual models as well as with their combinations according to the min-var(δ, η) criterion. The Table reports the results for each of the 4 data sets (50SandP, 100Ind, 45Euro, and 17STI) described in Section 3.1. Asterisks denote the instances in which the portfolio standard deviation obtained with combined forecasts are statistically different at the 10% level with respect to the one obtained with the equally weighted combination min-var($\eta = 0$). We assume that optimal minimum variance portfolios are re-balanced on a daily basis.

[4] We performed extensive robustness checks regarding the choice of the block size. More specifically, we performed the test with the block size varying from 1 to 100. The results are very similar to those reported here. Moreover, we also performed the test of the statistical significance of differences in the variance of minimum variance portfolio returns. The results are also similar to those obtained with the standard deviation and are available upon request.

The most striking result in Table 1 is that all individual models delivered minimum variance portfolios with higher risk with respect to those obtained with the model combinations. This result is true for all data sets. First, when comparing the individual models with respect to benchmark, we observe that a simple model averaging is able to deliver less risky minimum variance portfolio with respect to the best single model in all cases, and that these differences are statistically significant. Second, when comparing among alternative model combinations, we find that varying the δ and η parameters bring improvements in many instances. We observe that many alternative model combinations outperform the simple model averaging. In general, lowering the value of δ (i.e. putting more weight on the recent performance of individual models) and increasing the value of η (i.e. putting more weight on the best single models) lead to improvement with respect to the min-var($\eta = 0$) specification in terms of portfolio risk. The best overall result in all data sets in terms of standard deviation of portfolio returns is achieved by the min-var(δ=0.85,η=10) combination. For instance, in the case of the 50SandP data set this specification delivered minimum variance portfolio returns with a standard deviation of 0.521, whereas the same figure for best individual model (ORE) is 0.648. In the case of the 100Ind data set, the min-var(δ=0.85,η=10) combination delivered minimum variance portfolio returns with a standard deviation of 0.403, whereas the same figure for best individual model (EWMA) is 0.562. In the case of the 45Euro data set, the min-var(δ=0.85,η=10) combination delivered minimum variance portfolio returns with a standard deviation of 0.643, whereas the same figure for best individual model (ORE) is 0.810. Finally, in the case of the 17STI data set, the min-var(δ=0.85,η=10) combination delivered minimum variance portfolio returns with a standard deviation of 0.584, whereas the same figure for best individual model (CCC) is 0.689.

The comparison of other performance statistics such as Sharpe ratios also suggest that min-var(δ, η) model combinations perform at least equivalently with respect to individual models. In the case of the 50SandP data set, for instance, the average Sharpe ratio across individual models is 0.092, whereas the same figure across all model combinations is 0.108. The corresponding figures for the other data sets are: 0.096 vs. 0.142 (100Ind), 0.054 vs. 0.077 (45Euro), and 0.046 vs. 0.050 (17STI). Moreover, the average portfolio turnover across model combinations is smaller with respect to individual models. In the case of the 50SandP data set, the average portfolio turnover is 0.265 whereas for the individual models is 0.447. The corresponding figures for the other data sets are: 0.754 vs. 1.273 (100Ind), 0.353 vs. 0.641 (45Euro), and 0.137 vs. 0.264 (17STI). These numbers suggest that the proposed model combinations is able to not deliver less risky portfolio but also attractive risk-adjusted returns and portfolio turnover that are at

least equivalent to those obtained by single models.

Finally, we report in Table 2 the average and standard deviation of combination weights for each min-var(δ, η) specification. As expected, we observe that the best performing models as reported in Table 1 receive more weights in the model combination. This result is very much evident specially for the EWMA, ORE, and CCC specifications as they have weights higher than 20% in many instances.

3.4 Robustness checks

The results reported in Table 1 are based on optimal minimum variance portfolios re-balanced on daily basis. The transaction costs involved in this re-balancing frequency, however, might degrade the performance of the portfolios and hinder its implementation in practice. Thus, the performance of optimized portfolios is also evaluated in the case of weekly and monthly re-balancing frequencies. A potentially negative effect of adopting a lower re-balancing frequency is that the optimal compositions may become outdated.

As expected, we find that lowering the portfolio re-balancing frequency to a weekly and monthly basis leads to a substantial reduction in portfolio turnover. We also find that lowering the portfolio re-balancing has as impact in terms of higher portfolio standard deviation, since both individual models and their combinations displayed increased portfolio risk after lowering the re-balancing frequency. More importantly, however, is that several model combinations outperform individual models in terms of lower portfolio risk when the portfolios are re-balanced on a weekly and monthly basis for each of the 4 data sets. Therefore, the most important message of these robustness checks is that the outperformance of min-var(δ, η) combinations holds even a lower portfolio re-balancing setting.

4 Concluding remarks

Modeling and forecasting the covariance matrix of portfolio returns is of paramount importance in many economic and financial problems such as asset pricing, portfolio optimization and market risk management. In practice, however, the investor faces uncertainty on which is the most appropriate specification to model and to perform these tasks. To alleviate this problem, we put forward a novel approach to combine multivariate volatility predictions from alternative conditional covariance models. The proposed combination rule is explicitly built to exploit model complementarities and is motivated by the fact this class of models is often applied in portfolio selection problems. Four major advantages of this approach

are that i) does not require a proxy for the latent conditional covariance matrix, ii) does not require optimization of the combination weights, iii) holds the equally-weighted model combination as a particular case, and iv) accommodates alternative portfolio policies as well as alternative portfolio characteristics for defining the mixing. Our empirical evidence based on 4 data sets with cross section dimensions ranging from 17 to 100 assets over a 10-year time span confirm that minimum variance portfolios obtained with the combined conditional covariance estimators are substantially and statistically less risky with respect to all individual models considered.

Table 1: **Minimum variance portfolio performance: individual models vs. model combinations**

The Table reports performance statistics for minimum variance portfolios obtained with a set of individual models as well as with model combinations based on the min-var(δ, η) combination rule in (8) for $\delta = \{1, 0.95, 0.85\}$ and $\eta = \{0, 1, 2, 4, 10\}$. The individual models are the following multivariate GARCH specifications: exponentially weighted moving average (EWMA), optimal rolling estimator (ORE), VECH, orthogonal GARCH (O-GARCH), constant conditional correlation (CCC), dynamic conditional correlation (DCC), dynamic equicorrelation (DECO), and asymmetric DCC (ASYDCC). The figures are based on daily observations of 4 data sets from the global stock markets from 01/01/2004 until 31/12/2013. The first data set consists of closing prices of the 50 mostly traded stocks belonging to the US stock market index SandP100 (50SandP). The second data set consists of 100 US industry portfolios (100Ind) obtained from Ken French's web site. The third data set consists of closing prices of 45 stocks belonging to the European stock market index Eurostoxx (45Euro). Finally, the fourth data sets consists of closing prices of 17 stocks belonging to the Asian stock market index STI (17STI). All figures are based on out-of-sample daily observations. Optimal portfolios are re-balanced on a daily basis. Asterisks indicate that the standard deviation of portfolio returns is statistically different than that obtained with the equally weighted model combination min-var$(\eta = 0)$ at a significance level of 10%.

	Mean return (%)	Std. dev. (%)	Turnover	Sharpe ratio	Mean return (%)	Std. dev. (%)	Turnover	Sharpe ratio	Mean return (%)	Std. dev. (%)	Turnover	Sharpe ratio	Mean return (%)	Std. dev. (%)	Turnover	Sharpe ratio
	Data set: 50SandP				**Data set: 100Ind**				**Data set: 45Euro**				**Data set: 17STI**			
Individual models																
EWMA	0.071	0.687*	0.296	0.103	0.080	0.562*	0.680	0.143	0.060	0.814*	0.212	0.074	0.033	0.705*	0.060	0.046
ORE	0.064	0.648*	0.089	0.099	0.069	0.631*	0.075	0.110	0.057	0.810*	0.100	0.070	0.033	0.707*	0.056	0.047
VECH	0.077	0.726*	0.473	0.106	0.077	0.606*	1.265	0.127	0.054	0.843*	0.549	0.064	0.033	0.703*	0.207	0.047
O-GARCH	0.065	0.690*	0.051	0.094	0.055	0.728*	0.090	0.076	0.032	0.952*	0.101	0.033	0.031	0.727*	0.050	0.042
CCC	0.060	0.675*	0.403	0.088	0.065	0.581*	1.139	0.112	0.060	0.815*	0.478	0.074	0.042	0.689*	0.205	0.061
DCC	0.049	1.237*	1.828	0.039	0.044	1.011*	4.599	0.044	0.004	1.460*	2.734	0.002	0.042	1.154*	1.214	0.037
DECO	0.069	0.707*	0.264	0.097	0.057	0.674*	0.629	0.085	0.044	0.909*	0.399	0.048	0.023	0.732*	0.171	0.031
ASY-DCC	0.077	0.682*	0.410	0.112	0.058	0.829*	1.704	0.070	0.056	0.839*	0.556	0.067	0.039	0.699*	0.149	0.056
Model combinations																
min-var(η=0) (benchmark)	0.066	0.625	0.263	0.106	0.069	0.506	0.675	0.136	0.054	0.759	0.346	0.072	0.035	0.671	0.146	0.052
min-var(δ=1,η=1)	0.066	0.622	0.236	0.106	0.070	0.510*	0.586	0.137	0.056	0.757	0.325	0.074	0.036	0.670	0.122	0.053
min-var(δ=1,η=2)	0.066	0.621*	0.226	0.106	0.070	0.515*	0.540	0.137	0.057	0.756	0.317	0.076	0.036	0.670	0.114	0.054
min-var(δ=1,η=4)	0.065	0.622	0.228	0.105	0.071	0.523*	0.514	0.136	0.059	0.757	0.321	0.079	0.037	0.669	0.113	0.055
min-var(δ=1,η=10)	0.065	0.625	0.257	0.105	0.073	0.541*	0.526	0.135	0.062	0.769	0.371	0.081	0.038	0.670	0.126	0.057
min-var(δ=0.95,η=1)	0.067	0.613*	0.229	0.108	0.070	0.493*	0.633	0.142	0.056	0.748*	0.312	0.075	0.034	0.662*	0.124	0.052
min-var(δ=0.95,η=2)	0.067	0.605*	0.221	0.110	0.071	0.483*	0.645	0.147	0.056	0.739*	0.306	0.076	0.034	0.656*	0.117	0.051
min-var(δ=0.95,η=4)	0.066	0.593*	0.230	0.112	0.070	0.467*	0.752	0.150	0.057	0.726*	0.318	0.078	0.032	0.647*	0.117	0.050
min-var(δ=0.95,η=10)	0.065	0.575*	0.280	0.112	0.063	0.457*	1.071	0.139	0.057	0.706*	0.376	0.081	0.029	0.632*	0.137	0.046
min-var(δ=0.85,η=1)	0.065	0.600*	0.249	0.108	0.070	0.479*	0.707	0.146	0.057	0.731*	0.332	0.079	0.033	0.651*	0.131	0.051
min-var(δ=0.85,η=2)	0.064	0.579*	0.273	0.110	0.071	0.456*	0.859	0.155	0.059	0.710*	0.361	0.083	0.031	0.636*	0.136	0.048
min-var(δ=0.85,η=4)	0.061	0.552*	0.344	0.111	0.069	0.423*	1.231	0.164	0.060	0.680*	0.442	0.088	0.026	0.614*	0.161	0.043
min-var(δ=0.85,η=10)	0.058	0.521*	0.512	0.111	0.064	0.403*	1.853	0.158	0.056	0.643*	0.620	0.088	0.019	0.584*	0.227	0.033

Table 2: **Combination weights**

The Table reports the average and the standard deviation of the min-var(δ, η) combination weights for $\delta = \{1, 0.95, 0.85\}$ and $\eta = \{1, 2, 4, 10\}$. The individual models are the following multivariate GARCH specifications: exponentially weighted moving average (EWMA), optimal rolling estimator (ORE), VECH, orthogonal GARCH (O-GARCH), constant conditional correlation (CCC), dynamic conditional correlation (DCC), dynamic equicorrelation (DECO), and asymmetric DCC (ASYDCC). The figures are based on daily observations of 4 data sets from the global stock markets from 01/01/2004 until 31/12/2013. The first data set consists of closing prices of the 50 mostly traded stocks belonging to the US stock market index SandP100 (50SandP). The second data set consists of 100 US industry portfolios (100Ind) obtained from Ken French's web site. The third data set consists of closing prices of 45 stocks belonging to the European stock market index Eurostoxx (45Euro). Finally, the fourth data sets consists of closing prices of 17 stocks belonging to the Asian stock market index STI (17STI). All figures are based on out-of-sample daily observations.

	EWMA		ORE		VECH		OGARCH		CCC		DCC		DECO		ASYDCC	
	Avg. weight	Std. dev.	Avg. weight	Std. dev.	Avg. weight	Std. dev.	Avg. weight	Std. dev.	Avg. weight	Std. dev.	Avg. weight	Std. dev.	Avg. weight	Std. dev.	Avg. weight	Std. dev.
Data set: 50SandP																
min-var(δ=1,η=1)	0.135	0.000	0.148	0.001	0.099	0.002	0.124	0.002	0.154	0.001	0.053	0.002	0.139	0.001	0.149	0.001
min-var(δ=1,η=2)	0.136	0.001	0.165	0.002	0.074	0.003	0.115	0.003	0.178	0.002	0.021	0.001	0.144	0.001	0.167	0.003
min-var(δ=1,η=4)	0.128	0.002	0.187	0.005	0.037	0.003	0.092	0.005	0.219	0.005	0.003	0.000	0.143	0.002	0.192	0.007
min-var(δ=1,η=10)	0.083	0.004	0.216	0.020	0.004	0.001	0.037	0.006	0.319	0.014	0.000	0.000	0.111	0.004	0.230	0.016
min-var(δ=0.95,η=1)	0.135	0.018	0.149	0.020	0.123	0.021	0.140	0.028	0.145	0.018	0.045	0.013	0.127	0.020	0.136	0.017
min-var(δ=0.95,η=2)	0.137	0.036	0.166	0.044	0.114	0.039	0.149	0.059	0.157	0.038	0.016	0.010	0.122	0.037	0.139	0.036
min-var(δ=0.95,η=4)	0.133	0.065	0.192	0.099	0.096	0.065	0.163	0.119	0.170	0.079	0.002	0.004	0.107	0.066	0.136	0.073
min-var(δ=0.95,η=10)	0.115	0.120	0.244	0.240	0.066	0.115	0.187	0.221	0.184	0.180	0.000	0.000	0.079	0.113	0.125	0.155
min-var(δ=0.85,η=1)	0.134	0.031	0.147	0.033	0.124	0.034	0.141	0.044	0.145	0.031	0.048	0.024	0.126	0.033	0.136	0.030
min-var(δ=0.85,η=2)	0.134	0.059	0.161	0.068	0.118	0.065	0.152	0.091	0.155	0.065	0.021	0.025	0.120	0.063	0.139	0.063
min-var(δ=0.85,η=4)	0.128	0.104	0.177	0.135	0.107	0.114	0.169	0.169	0.165	0.128	0.007	0.026	0.109	0.110	0.138	0.123
min-var(δ=0.85,η=10)	0.113	0.180	0.194	0.254	0.092	0.189	0.189	0.278	0.176	0.247	0.004	0.040	0.092	0.179	0.139	0.230
Data set: 100Ind																
min-var(δ=1,η=1)	0.149	0.006	0.187	0.015	0.135	0.005	0.126	0.004	0.183	0.002	0.060	0.001	0.112	0.003	0.047	0.004
min-var(δ=1,η=2)	0.155	0.015	0.244	0.035	0.128	0.011	0.110	0.005	0.234	0.004	0.025	0.001	0.088	0.007	0.016	0.003
min-var(δ=1,η=4)	0.139	0.032	0.340	0.080	0.094	0.020	0.068	0.003	0.310	0.022	0.004	0.001	0.044	0.008	0.001	0.001
min-var(δ=1,η=10)	0.064	0.037	0.490	0.185	0.023	0.013	0.009	0.002	0.410	0.133	0.000	0.000	0.004	0.002	0.000	0.000
min-var(δ=0.95,η=1)	0.170	0.030	0.131	0.023	0.143	0.025	0.114	0.025	0.173	0.041	0.055	0.019	0.124	0.023	0.089	0.035
min-var(δ=0.95,η=2)	0.209	0.071	0.125	0.044	0.149	0.052	0.096	0.042	0.219	0.095	0.024	0.017	0.112	0.042	0.065	0.059
min-var(δ=0.95,η=4)	0.271	0.159	0.099	0.076	0.139	0.093	0.065	0.061	0.297	0.205	0.005	0.008	0.082	0.067	0.041	0.087
min-var(δ=0.95,η=10)	0.369	0.325	0.056	0.131	0.090	0.128	0.030	0.076	0.394	0.367	0.000	0.000	0.038	0.090	0.023	0.101
min-var(δ=0.85,η=1)	0.165	0.047	0.129	0.035	0.143	0.044	0.115	0.040	0.175	0.062	0.058	0.029	0.124	0.043	0.092	0.049
min-var(δ=0.85,η=2)	0.196	0.104	0.119	0.066	0.148	0.089	0.100	0.073	0.221	0.137	0.029	0.031	0.114	0.082	0.072	0.084
min-var(δ=0.85,η=4)	0.233	0.207	0.095	0.110	0.147	0.156	0.079	0.123	0.284	0.258	0.010	0.028	0.095	0.134	0.056	0.126
min-var(δ=0.85,η=10)	0.262	0.340	0.070	0.175	0.136	0.239	0.066	0.187	0.342	0.391	0.003	0.031	0.072	0.190	0.049	0.172
Data set: 45Euro																
min-var(δ=1,η=1)	0.132	0.002	0.139	0.002	0.125	0.002	0.108	0.002	0.163	0.002	0.067	0.004	0.125	0.001	0.142	0.001
min-var(δ=1,η=2)	0.133	0.004	0.147	0.004	0.119	0.003	0.089	0.002	0.204	0.004	0.034	0.005	0.120	0.002	0.154	0.001
min-var(δ=1,η=4)	0.125	0.007	0.153	0.007	0.099	0.005	0.056	0.003	0.292	0.013	0.008	0.002	0.101	0.003	0.166	0.003
min-var(δ=1,η=10)	0.071	0.012	0.116	0.018	0.040	0.007	0.009	0.001	0.582	0.038	0.000	0.000	0.041	0.002	0.141	0.007
min-var(δ=0.95,η=1)	0.148	0.021	0.149	0.021	0.139	0.019	0.111	0.029	0.149	0.025	0.047	0.019	0.118	0.022	0.139	0.015
min-var(δ=0.95,η=2)	0.162	0.043	0.164	0.042	0.143	0.039	0.096	0.048	0.167	0.054	0.019	0.017	0.106	0.039	0.143	0.030
min-var(δ=0.95,η=4)	0.179	0.091	0.182	0.084	0.143	0.076	0.076	0.069	0.194	0.120	0.005	0.010	0.082	0.063	0.139	0.058
min-var(δ=0.95,η=10)	0.202	0.205	0.212	0.194	0.134	0.163	0.053	0.102	0.244	0.263	0.000	0.001	0.044	0.091	0.111	0.118
min-var(δ=0.85,η=1)	0.145	0.032	0.147	0.032	0.137	0.030	0.113	0.040	0.151	0.041	0.050	0.029	0.120	0.038	0.138	0.029
min-var(δ=0.85,η=2)	0.155	0.066	0.159	0.064	0.139	0.061	0.101	0.068	0.170	0.087	0.024	0.030	0.112	0.072	0.140	0.058
min-var(δ=0.85,η=4)	0.163	0.127	0.170	0.121	0.136	0.115	0.090	0.111	0.197	0.169	0.010	0.028	0.100	0.122	0.134	0.109
min-var(δ=0.85,η=10)	0.166	0.232	0.176	0.228	0.131	0.213	0.083	0.188	0.233	0.300	0.004	0.023	0.085	0.189	0.122	0.201
Data set: 17STI																
min-var(δ=1,η=1)	0.134	0.000	0.135	0.000	0.128	0.001	0.132	0.000	0.150	0.001	0.048	0.000	0.134	0.001	0.140	0.000
min-var(δ=1,η=2)	0.136	0.001	0.138	0.001	0.124	0.002	0.131	0.001	0.170	0.001	0.017	0.000	0.136	0.002	0.148	0.000
min-var(δ=1,η=4)	0.132	0.002	0.137	0.002	0.109	0.003	0.123	0.001	0.206	0.003	0.002	0.000	0.133	0.003	0.156	0.001
min-var(δ=1,η=10)	0.110	0.004	0.119	0.004	0.068	0.005	0.092	0.003	0.332	0.009	0.000	0.000	0.112	0.005	0.167	0.002
min-var(δ=0.95,η=1)	0.135	0.015	0.134	0.016	0.138	0.013	0.132	0.022	0.141	0.015	0.057	0.021	0.125	0.020	0.138	0.012
min-var(δ=0.95,η=2)	0.138	0.029	0.137	0.030	0.144	0.027	0.135	0.043	0.151	0.032	0.028	0.024	0.121	0.039	0.145	0.026
min-var(δ=0.95,η=4)	0.138	0.053	0.137	0.055	0.149	0.055	0.137	0.081	0.166	0.067	0.010	0.020	0.112	0.071	0.150	0.059
min-var(δ=0.95,η=10)	0.130	0.101	0.129	0.105	0.149	0.129	0.146	0.170	0.200	0.168	0.003	0.018	0.099	0.137	0.146	0.141
min-var(δ=0.85,η=1)	0.134	0.025	0.134	0.026	0.137	0.022	0.131	0.031	0.140	0.026	0.061	0.031	0.126	0.037	0.138	0.023
min-var(δ=0.85,η=2)	0.136	0.048	0.136	0.049	0.142	0.046	0.132	0.060	0.149	0.054	0.036	0.038	0.126	0.074	0.143	0.048
min-var(δ=0.85,η=4)	0.134	0.084	0.134	0.087	0.144	0.094	0.133	0.111	0.160	0.109	0.019	0.047	0.129	0.140	0.147	0.098
min-var(δ=0.85,η=10)	0.123	0.138	0.127	0.147	0.147	0.200	0.135	0.208	0.175	0.229	0.012	0.075	0.140	0.251	0.141	0.188

Table 3: **Minimum variance portfolio performance: individual models vs. model combinations under weekly re-balancing**

The Table reports performance statistics for minimum variance portfolios obtained with a set of individual models as well as with model combinations based on the min-var(δ, η) combination rule in (8) for $\delta = \{1, 0.95, 0.85\}$ and $\eta = \{0, 1, 2, 4, 10\}$. The individual models are the following multivariate GARCH specifications: exponentially weighted moving average (EWMA), optimal rolling estimator (ORE), VECH, orthogonal GARCH (O-GARCH), constant conditional correlation (CCC), dynamic conditional correlation (DCC), dynamic equicorrelation (DECO), and asymmetric DCC (ASYDCC). The figures are based on daily observations of 4 data sets from the global stock markets from 01/01/2004 until 31/12/2013. The first data set consists of closing prices of the 50 mostly traded stocks belonging to the US stock market index SandP100 (50SandP). The second data set consists of 100 US industry portfolios (100Ind) obtained from Ken French's web site. The third data set consists of closing prices of 45 stocks belonging to the European stock market index Eurostoxx (45Euro). Finally, the fourth data sets consists of closing prices of 17 stocks belonging to the Asian stock market index STI (17STI). All figures are based on out-of-sample daily observations. Optimal portfolios are re-balanced on a weekly basis. Asterisks indicate that the standard deviation of portfolio returns is statistically different than that obtained with the equally weighted model combination min-var$(\eta = 0)$ at a significance level of 10%.

	Mean return (%)	Std. dev. (%)	Turnover	Sharpe ratio	Mean return (%)	Std. dev. (%)	Turnover	Sharpe ratio	Mean return (%)	Std. dev. (%)	Turnover	Sharpe ratio	Mean return (%)	Std. dev. (%)	Turnover	Sharpe ratio
	Data set: 50SandP				**Data set: 100Ind**				**Data set: 45Euro**				**Data set: 17STI**			
Individual models																
EWMA	0.071	0.686*	0.180	0.103	0.083	0.569*	0.430	0.145	0.061	0.820*	0.142	0.075	0.034	0.712*	0.043	0.048
ORE	0.063	0.648*	0.065	0.098	0.070	0.670*	0.071	0.105	0.058	0.813*	0.075	0.071	0.034	0.714*	0.040	0.048
VECH	0.080	0.725*	0.275	0.111	0.083	0.615*	0.750	0.135	0.066	0.840*	0.325	0.079	0.035	0.719*	0.119	0.049
O-GARCH	0.066	0.689*	0.038	0.096	0.055	0.732*	0.071	0.075	0.033	0.944*	0.072	0.035	0.030	0.728*	0.032	0.042
CCC	0.056	0.673*	0.216	0.083	0.063	0.589*	0.594	0.107	0.056	0.813*	0.249	0.069	0.038	0.712*	0.111	0.054
DCC	0.064	1.260*	0.925	0.051	0.060	1.008*	2.456	0.059	0.101	1.499*	1.370	0.068	0.042	1.189*	0.574	0.036
DECO	0.068	0.707*	0.146	0.097	0.059	0.678*	0.344	0.087	0.037	0.915*	0.215	0.040	0.022	0.752*	0.093	0.029
ASY-DCC	0.078	0.690*	0.228	0.112	0.064	0.840*	0.997	0.077	0.068	0.832*	0.299	0.082	0.038	0.715*	0.085	0.053
Model combinations																
min-var(η=0) (benchmark)	0.066	0.627	0.147	0.106	0.070	0.519	0.393	0.136	0.057	0.755	0.191	0.075	0.037	0.683	0.082	0.054
min-var(δ=1,η=1)	0.065	0.625*	0.132	0.105	0.071	0.524*	0.345	0.135	0.057	0.753	0.179	0.076	0.036	0.682	0.070	0.053
min-var(δ=1,η=2)	0.065	0.624*	0.127	0.104	0.071	0.529*	0.320	0.134	0.058	0.753	0.174	0.076	0.036	0.682	0.066	0.053
min-var(δ=1,η=4)	0.064	0.625	0.128	0.102	0.071	0.539*	0.305	0.132	0.058	0.753	0.176	0.077	0.036	0.682	0.066	0.053
min-var(δ=1,η=10)	0.063	0.628	0.142	0.101	0.072	0.560*	0.306	0.128	0.059	0.765	0.198	0.077	0.037	0.684	0.072	0.054
min-var(δ=0.95,η=1)	0.066	0.622*	0.129	0.106	0.072	0.515*	0.368	0.139	0.056	0.753	0.174	0.074	0.035	0.679*	0.072	0.052
min-var(δ=0.95,η=2)	0.067	0.620*	0.124	0.108	0.072	0.513*	0.370	0.141	0.055	0.752	0.171	0.074	0.034	0.678*	0.069	0.051
min-var(δ=0.95,η=4)	0.068	0.619*	0.127	0.110	0.071	0.514	0.418	0.139	0.054	0.752	0.176	0.072	0.033	0.676*	0.070	0.049
min-var(δ=0.95,η=10)	0.070	0.621	0.149	0.112	0.065	0.535	0.563	0.122	0.055	0.755	0.201	0.073	0.030	0.676*	0.080	0.044
min-var(δ=0.85,η=1)	0.066	0.620*	0.136	0.107	0.072	0.513*	0.391	0.141	0.056	0.751*	0.181	0.075	0.034	0.677*	0.076	0.051
min-var(δ=0.85,η=2)	0.067	0.616*	0.143	0.109	0.073	0.511*	0.443	0.143	0.055	0.751	0.189	0.074	0.032	0.674*	0.078	0.048
min-var(δ=0.85,η=4)	0.069	0.614*	0.172	0.112	0.073	0.515	0.597	0.142	0.055	0.754	0.220	0.072	0.028	0.670*	0.088	0.042
min-var(δ=0.85,η=10)	0.070	0.621	0.242	0.113	0.075	0.543*	0.844	0.139	0.056	0.766	0.292	0.073	0.021	0.671*	0.116	0.031

Table 4: **Minimum variance portfolio performance: individual models vs. model combinations under monthly re-balancing**

The Table reports performance statistics for minimum variance portfolios obtained with a set of individual models as well as with model combinations based on the min-var(δ, η) combination rule in (8) for $\delta = \{1, 0.95, 0.85\}$ and $\eta = \{0, 1, 2, 4, 10\}$. The individual models are the following multivariate GARCH specifications: exponentially weighted moving average (EWMA), optimal rolling estimator (ORE), VECH, orthogonal GARCH (O-GARCH), constant conditional correlation (CCC), dynamic conditional correlation (DCC), dynamic equicorrelation (DECO), and asymmetric DCC (ASYDCC). The figures are based on daily observations of 4 data sets from the global stock markets from 01/01/2004 until 31/12/2013. The first data set consists of closing prices of the 50 mostly traded stocks belonging to the US stock market index SandP100 (50SandP). The second data set consists of 100 US industry portfolios (100Ind) obtained from Ken French's web site. The third data set consists of closing prices of 45 stocks belonging to the European stock market index Eurostoxx (45Euro). Finally, the fourth data sets consists of closing prices of 17 stocks belonging to the Asian stock market index STI (17STI). All figures are based on out-of-sample daily observations. Optimal portfolios are re-balanced on a monthly basis. Asterisks indicate that the standard deviation of portfolio returns is statistically different than that obtained with the equally weighted model combination min-var$(\eta = 0)$ at a significance level of 10%.

	Mean return (%)	Std. dev. (%)	Turnover	Sharpe ratio	Mean return (%)	Std. dev. (%)	Turnover	Sharpe ratio	Mean return (%)	Std. dev. (%)	Turnover	Sharpe ratio	Mean return (%)	Std. dev. (%)	Turnover	Sharpe ratio
	Data set: 50SandP				**Data set: 100Ind**				**Data set: 45Euro**				**Data set: 17STI**			
Individual models																
EWMA	0.075	0.697*	0.107	0.107	0.080	0.590*	0.265	0.135	0.058	0.841*	0.096	0.069	0.029	0.724*	0.030	0.041
ORE	0.064	0.653*	0.047	0.098	0.071	0.690*	0.065	0.103	0.055	0.828*	0.059	0.067	0.030	0.726*	0.029	0.041
VECH	0.082	0.731*	0.154	0.113	0.080	0.623*	0.425	0.128	0.068	0.856*	0.182	0.080	0.027	0.735*	0.060	0.037
O-GARCH	0.061	0.705*	0.026	0.086	0.056	0.741*	0.054	0.076	0.034	0.948*	0.050	0.036	0.028	0.728*	0.021	0.039
CCC	0.063	0.667*	0.099	0.095	0.065	0.605*	0.269	0.107	0.044	0.806*	0.114	0.055	0.033	0.717*	0.050	0.046
DCC	0.051	1.225*	0.433	0.041	0.050	1.034*	1.249	0.048	0.123	1.594*	0.655	0.077	0.023	1.218*	0.226	0.019
DECO	0.068	0.709*	0.070	0.096	0.052	0.682*	0.166	0.077	0.029	0.895*	0.104	0.032	0.012	0.763*	0.043	0.016
ASY-DCC	0.080	0.693*	0.115	0.116	0.062	0.832*	0.566	0.075	0.062	0.847*	0.155	0.074	0.029	0.727*	0.043	0.040
Model combinations																
min-var(η=0) (benchmark)	0.069	0.629	0.073	0.109	0.073	0.541	0.210	0.135	0.052	0.766	0.097	0.068	0.028	0.695	0.040	0.041
min-var(δ=1,η=1)	0.068	0.626*	0.066	0.108	0.073	0.546*	0.186	0.133	0.053	0.765	0.092	0.069	0.028	0.693	0.035	0.041
min-var(δ=1,η=2)	0.067	0.625*	0.063	0.108	0.073	0.552*	0.174	0.132	0.053	0.764	0.089	0.070	0.029	0.693	0.034	0.041
min-var(δ=1,η=4)	0.067	0.625*	0.063	0.107	0.073	0.562*	0.166	0.130	0.054	0.765	0.089	0.071	0.029	0.693	0.034	0.041
min-var(δ=1,η=10)	0.068	0.626	0.069	0.108	0.073	0.583*	0.162	0.125	0.053	0.773	0.096	0.069	0.029	0.694	0.036	0.042
min-var(δ=0.95,η=1)	0.069	0.626*	0.065	0.110	0.073	0.540	0.197	0.136	0.051	0.766	0.090	0.067	0.028	0.693*	0.036	0.040
min-var(δ=0.95,η=2)	0.070	0.627	0.063	0.111	0.073	0.541	0.197	0.136	0.051	0.768	0.090	0.066	0.028	0.692*	0.035	0.040
min-var(δ=0.95,η=4)	0.071	0.629	0.064	0.112	0.072	0.549	0.220	0.131	0.049	0.773	0.093	0.063	0.028	0.692*	0.036	0.040
min-var(δ=0.95,η=10)	0.071	0.637*	0.072	0.112	0.063	0.585*	0.281	0.107	0.046	0.785*	0.105	0.059	0.028	0.694	0.039	0.040
min-var(δ=0.85,η=1)	0.069	0.627	0.066	0.110	0.073	0.538	0.201	0.137	0.051	0.766	0.092	0.067	0.028	0.691*	0.037	0.040
min-var(δ=0.85,η=2)	0.069	0.627	0.066	0.111	0.073	0.537	0.213	0.137	0.050	0.770	0.095	0.065	0.027	0.689*	0.037	0.039
min-var(δ=0.85,η=4)	0.071	0.632	0.073	0.112	0.071	0.550	0.263	0.129	0.048	0.778	0.104	0.062	0.025	0.688*	0.040	0.037
min-var(δ=0.85,η=10)	0.075	0.645*	0.091	0.117	0.067	0.605*	0.339	0.112	0.050	0.792*	0.127	0.064	0.023	0.697	0.049	0.033

References

Aiolfi, Marco, and Timmermann, Allan. 2006. Persistence in forecasting performance and conditional combination strategies. *Journal of Econometrics*, **135**(1-2), 31–53.

Alexander, Carol. 2001. Orthogonal garch. *Mastering risk*, **2**, 21–38.

Amendola, Alessandra, and Storti, Giuseppe. 2015. Model uncertainty and forecast combination in high-dimensional multivariate volatility prediction. *Journal of Forecasting, forthcoming.*

Armstrong, J Scott. 2001. Combining forecasts. *Pages 417–439 of: Principles of forecasting.* Springer.

Asai, Manabu. 2013. Heterogeneous Asymmetric Dynamic Conditional Correlation Model with Stock Return and Range. *Journal of Forecasting*, **32**(5), 469–480.

Asai, Manabu, McAleer, Michael, and Yu, Jun. 2006. Multivariate stochastic volatility: a review. *Econometric Reviews*, **25**(2-3), 145–175.

Bauwens, L., Laurent, S., and Rombouts, J.V.K. 2006. Multivariate GARCH models: a survey. *Journal of Applied Econometrics*, **21**(1), 79–109.

Becker, Ralf, and Clements, Adam E. 2008. Are combination forecasts of SandP 500 volatility statistically superior? *International Journal of Forecasting*, **24**(1), 122–133.

Becker, Ralf, Clements, AE, Doolan, MB, and Hurn, AS. 2014. Selecting volatility forecasting models for portfolio allocation purposes. *International Journal of Forecasting, forthcoming.*

Billio, Monica, and Caporin, Massimiliano. 2009. A generalized dynamic conditional correlation model for portfolio risk evaluation. *Mathematics and Computers in Simulation*, **79**(8), 2566–2578.

Bollerslev, T. 1990. Modelling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model. *Review of Economics and Statistics*, **72**(3), 498–505.

Bollerslev, Tim, Engle, Robert F, and Wooldridge, Jeffrey Marc. 1988. A Capital Asset Pricing Model with Time-Varying Covariances. *Journal of Political Economy*, **96**(1), 116–31.

Cappiello, L., Engle, R.F., and Sheppard, K. 2006. Asymmetric dynamics in the correlations of global equity and bond returns. *Journal of Financial Econometrics*, **4**(4), 537–572.

Chib, S., Omori, Y., and Asai, M. 2009. *Multivariate stochastic volatility.* Springer Verlag.

Clemen, Robert T. 1989. Combining forecasts: A review and annotated bibliography. *International Journal of Forecasting*, **5**(4), 559–583.

Colacito, Riccardo, Engle, Robert F, and Ghysels, Eric. 2011. A component model for dynamic correlations. *Journal of Econometrics*, **164**(1), 45–59.

De Menezes, Lilian M, W Bunn, Derek, and Taylor, James W. 2000. Review of guidelines for the use of combined forecasts. *European Journal of Operational Research*, **120**(1), 190–204.

DeMiguel, Victor, Garlappi, Lorenzo, and Uppal, Raman. 2009. Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? *Review of Financial Studies*, **22**(5), 1915–1953.

Diebold, Francis X. 1989. Forecast combination and encompassing: Reconciling two divergent literatures. *International Journal of Forecasting*, **5**(4), 589–592.

Engle, R. 2002. Dynamic conditional correlation: a simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business and Economic Statistics*, **20**(3), 339–350.

Engle, R.F., Shephard, N., and Sheppard, K. 2008. Fitting vast dimensional time-varying covariance models. *Discussion Paper Series n.403, Department of Economics, University of Oxford.*

Engle, Robert, and Colacito, Riccardo. 2006. Testing and Valuing Dynamic Correlations for Asset Allocation. *Journal of Business and Economic Statistics*, **24**, 238–253.

Engle, Robert, and Kelly, Bryan. 2012. Dynamic Equicorrelation. *Journal of Business and Economic Statistics*, **30**(2), 212–228.

Engle, Robert, and Mezrich, Joseph. 1996. GARCH for Groups. *Risk*, **9**, 36–40.

Engle, Robert, and Sheppard, Kevin. 2008. Evaluating the specification of covariance models for large portfolios. *New York University, working paper.*

Fleming, Jeff, Kirby, Chris, and Ostdiek, Barbara. 2001. The economic value of volatility timing. *The Journal of Finance*, **56**(1), 329–352.

Fleming, Jeff, Kirby, Chris, and Ostdiek, Barbara. 2003. The economic value of volatility timing using ŞrealizedŤ volatility. *Journal of Financial Economics*, **67**(3), 473–509.

Foster, Dean P, and Nelson, Daniel B. 1996. Continuous Record Asymptotics for Rolling Sample Variance Estimators. *Econometrica*, **64**(1), 139–74.

Genre, Véronique, Kenny, Geoff, Meyler, Aidan, and Timmermann, Allan. 2013. Combining expert forecasts: Can anything beat the simple average? *International Journal of Forecasting*, **29**(1), 108–121.

Geweke, John, and Amisano, Gianni. 2011. Optimal prediction pools. *Journal of Econometrics*, **164**(1), 130–141.

Granger, Clive W.J. 1989. Combining Forecasts–Twenty Years Later. *Journal of Forecasting*, **8**(3), 167–173.

Hafner, Christian M, and Franses, Philip Hans. 2009. A generalized dynamic conditional correlation model: simulation and application to many assets. *Econometric Reviews*, **28**(6), 612–631.

Hafner, Christian M, and Reznikova, Olga. 2012. On the estimation of dynamic conditional correlation models. *Computational Statistics and Data Analysis*, **56**(11), 3533–3545.

Hafner, Christian M, Van Dijk, Dick, Franses, Philip Hans, *et al.* 2006. Semiparametric modelling of correlation dynamics. *Advances in Econometrics*, **20**(Part A), 59–103.

Hansen, Peter Reinhard, Lunde, Asger, and Voev, Valeri. 2014. Realized beta garch: A multivariate garch model with realized measures of volatility. *Journal of Applied Econometrics*, **29**(5), 774–799.

Hendry, David F, and Clements, Michael P. 2004. Pooling of forecasts. *The Econometrics Journal*, **7**(1), 1–31.

Hibon, Michele, and Evgeniou, Theodoros. 2005. To combine or not to combine: selecting among forecasts and their combinations. *International Journal of Forecasting*, **21**(1), 15–24.

Jagannathan, R., and Ma, T. 2003. Risk reduction in large portfolios: Why imposing the wrong constraints helps. *The Journal of Finance*, **58**(4), 1651–1684.

Kang, Heejoon. 1986. Unstable weights in the combination of forecasts. *Management Science*, **32**(6), 683–695.

Kirby, Chris, and Ostdiek, Barbara. 2012. ItŠs all in the timing: simple active portfolio strategies that outperform naive diversification. *Journal of Financial and Quantitative Analysis*, **47**(02), 437–467.

Laurent, Sébastien, Rombouts, Jeroen VK, and Violante, Francesco. 2012. On the forecasting accuracy of multivariate GARCH models. *Journal of Applied Econometrics*, **27**(6), 934–955.

Laurent, Sébastien, Rombouts, Jeroen VK, and Violante, Francesco. 2013. On loss functions and ranking forecasting performances of multivariate volatility models. *Journal of Econometrics*, **173**(1), 1–10.

Ledoit, O., and Wolf, M. 2008. Robust performance hypothesis testing with the Sharpe ratio. *Journal of Empirical Finance*, **15**(5), 850–859.

Markowitz, H. 1952. Portfolio selection. *Journal of Finance*, **7**(1), 77–91.

Newbold, P., and Harvey, I. H. 2002. *Forecast combination and encompassing.* In M. P. Clements, and D. F. Hendry (Eds.), A companion to economic forecasting. Blackwell Publishers.

Noureldin, Diaa, Shephard, Neil, and Sheppard, Kevin. 2012. Multivariate high-frequency-based volatility (HEAVY) models. *Journal of Applied Econometrics*, **27**(6), 907–933.

Noureldin, Diaa, Shephard, Neil, and Sheppard, Kevin. 2014. Multivariate rotated ARCH models. *Journal of Econometrics*, **179**(1), 16–30.

Palm, Franz C, and Zellner, Arnold. 1992. To combine or not to combine? Issues of combining forecasts. *Journal of Forecasting*, **11**(8), 687–701.

Patton, Andrew J, and Sheppard, Kevin. 2009. Optimal combinations of realised volatility estimators. *International Journal of Forecasting*, **25**(2), 218–238.

Pelletier, Denis. 2006. Regime switching for dynamic correlations. *Journal of econometrics*, **131**(1), 445–473.

Pesaran, M. Hashem, and Timmermann, Allan. 2007. Selection of estimation window in the presence of breaks. *Journal of Econometrics*, **137**(1), 134–161.

Philipov, Alexander, and Glickman, Mark E. 2006a. Factor multivariate stochastic volatility via Wishart processes. *Econometric Reviews*, **25**(2-3), 311–334.

Philipov, Alexander, and Glickman, Mark E. 2006b. Multivariate stochastic volatility via Wishart processes. *Journal of Business and Economic Statistics*, **24**(3), 313–328.

Politis, D.N., and Romano, J.P. 1994. The stationary bootstrap. *Journal of the American Statistical Association*, **89**(428), 1303–1313.

Pooter, Michiel de, Martens, Martin, and Dijk, Dick van. 2008. Predicting the daily covariance matrix for sandp 100 stocks using intraday dataŮbut which frequency to use? *Econometric Reviews*, **27**(1-3), 199–229.

Rangel, José Gonzalo, and Engle, Robert F. 2012. The factor–spline–GARCH model for high and low frequency correlations. *Journal of Business and Economic Statistics*, **30**(1), 109–124.

Santos, André AP, and Moura, Guilherme V. 2014. Dynamic factor multivariate GARCH model. *Computational Statistics and Data Analysis*, **76**, 606–617.

Silvennoinen, A., and Teräsvirta, T. 2009. Multivariate GARCH models. *In:* Andersen, T.G., Davis, R.A., Kreiss, J.-P., and Mikosch, T. (eds), *Handbook of Financial Time Series.* Springer Verlag.

Silvennoinen, Annastiina, and Teräsvirta, Timo. 2015. Modeling conditional correlations of asset returns: A smooth transition approach. *Econometric Reviews*, **34**(1-2), 174–197.

Stock, James H, and Watson, Mark W. 2004. Combination forecasts of output growth in a seven-country data set. *Journal of Forecasting*, **23**(6), 405–430.

Timmermann, Allan. 2006. Forecast combinations. *Handbook of economic forecasting*, **1**, 135–196.

Wallis, Kenneth F. 2011. Combining forecasts–forty years later. *Applied Financial Economics*, **21**(1-2), 33–41.

Yang, Yuhong. 2004. Combining forecasting procedures: some theoretical results. *Econometric Theory*, **20**(01), 176–222.

Zaffaroni, Paolo. 2008. Large-scale volatility models: theoretical properties of professionalsŠ practice. *Journal of Time Series Analysis*, **29**(3), 581–599.

Zhang, Kun, and Chan, Laiwan. 2009. Efficient factor garch models and factor-dcc models. *Quantitative Finance*, **9**(1), 71–91.