

Volatility Risk Pass-Through

Ric Colacito



Max Croce



Yang Liu



Ivan Shaliastovich



Main Question

Uncertainty in a one-country setting:

- Sizeable impact of volatility risks on growth and asset prices
- Typically, high aggregate volatility is “bad”:
 - Lowers output and investment
 - Lowers asset valuations
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Open question:

- How are volatility risks shared internationally?
 - **Novel empirical investigation** on G17
 - **Novel theoretical insights** on volatility risk-sharing

Main Findings

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4. Explain these findings with a recursive risk sharing of output vol risks

Empirical Analysis

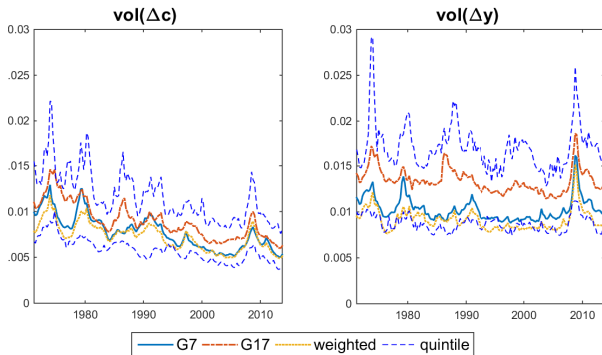
Empirical Analysis

- Quarterly data for 17 major industrialized countries from 1971 to 2014
- Output is consumption plus net exports
 - Abstract for now from investment and government expenditure
- For variable of interest in each country, run a filter:

$$z_t = \mu(1 - \rho) + \rho z_{t-1} + e^{\sigma_t(z)/2} \eta_t$$
$$\sigma_t(z) = \mu_\sigma(1 - \nu) + \nu \sigma_{t-1}(z) + \sigma_w w_t$$

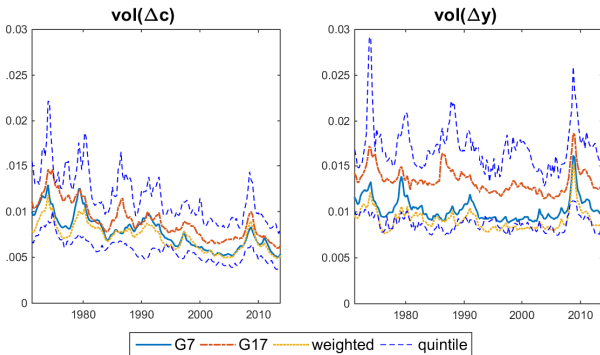
- z is real output, consumption, net exports, exchange rates
- $\sigma(z)$ is our estimate of the short-run volatility

Macroeconomic Volatilities



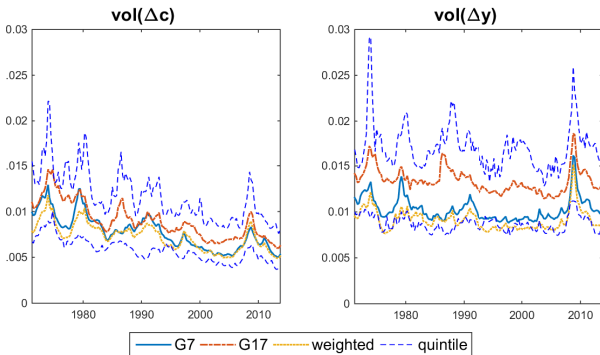
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Macroeconomic Volatilities



1. Substantial persistent movements in macro vols
2. Across countries: $\rho(\sigma_t^y, \sigma_t^{y*}) = 0.30 < \rho(\sigma_t^c, \sigma_t^{c*}) = 0.50$
3. Within countries: $\rho(\sigma_t^c, \sigma_t^y) = 0.70 < 1 \rightarrow$ international pass-through.

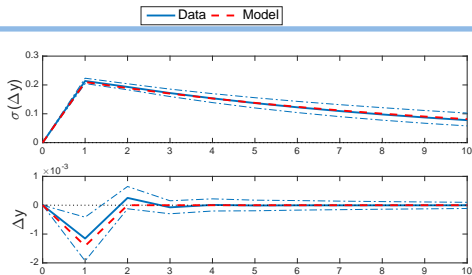
Measuring Relative Impulse Impact

- Identify impact of relative output vols on quantities
- In benchmark case, stack country variables, relative to US:

$$\tilde{Y}_{i,t} = \begin{bmatrix} \sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US}) \\ \Delta y_i - \Delta y_{US} \\ \sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}) \\ \Delta c_i - \Delta c_{US} \\ \Delta(NX/Y)_i - \Delta(NX/Y)_{US} \end{bmatrix},$$

- Estimate a pooled VAR(1) across countries
- Trace impulse response of relative output vol shocks on consumption, net exports, and consumption volatility

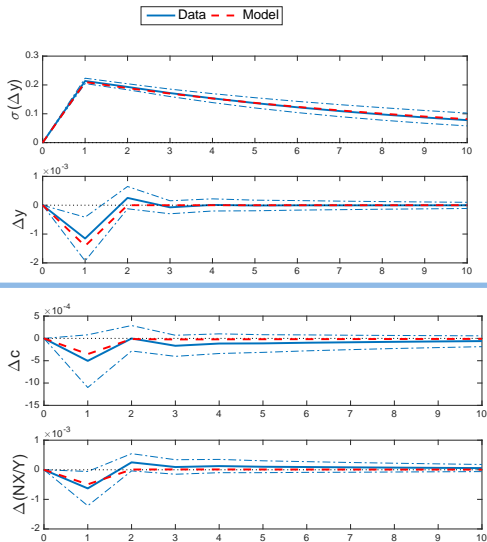
Response to Volatility Shocks



Take-aways:

- High output volatility decreases the growth rate of output

Response to Volatility Shocks



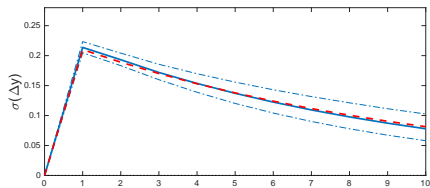
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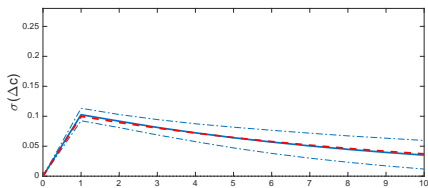
- However, net imports increase, and consumption falls by less
Evidence of international risk-sharing

Volatility Pass-Through

Output Vol



Consumption Vol



High Output Vol increases Consumption Vol less than one-to-one

Volatility Pass-Through Index (VPTI) ▶ Details

$$\text{Pass-through Index} := 1 - \frac{\partial(\sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}))}{\partial(\sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US}))}$$

- Interpretation of VPTI with one good and CRRA
 - 0 \rightarrow no risk sharing, i.e., autarky ($\Delta c_{i,t} = \Delta y_{i,t}$)
 - 1 \rightarrow perfect risk sharing ($\Delta c_{i,t} = \Delta c_{j,t}$)

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- In the data:
 - G7 countries, VPTI = 50%
 - Bottom-10 G17 countries, VPTI = 60%
 - Bottom-10 G17 countries, VPTI = 70% w.r.t. shocks originating in small countries

Volatility Disconnect Puzzle

- By no-arbitrage + CRRA, FX and consumption diff.s connected:

$$\Delta e_{t+1} = \gamma \times (\Delta c_{h,t+1} - \Delta c_{f,t+1})$$

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- This paper: empirical disconnect of **vols**

$$\text{Corr}(\text{Var}_t[\Delta e_{t+1}], \text{Var}_t[\Delta c_{h,t+1} - \Delta c_{f,t+1}]) \approx 0.20$$

- Puzzle with CRRA
- Puzzle for EZ models that address the Backus-Smith puzzle (among others, Colacito Croce (2011,2013))

Model

Model

- Two countries: home (h) and foreign (f)
- Recursive **EZ** utility over the consumption aggregate C_t

$$C_t^h = (x_t^h)^\alpha (y_t^h)^{1-\alpha}, \quad C_t^f = (x_t^f)^{1-\alpha} (y_t^f)^\alpha$$

- x^h , x^f , y^h , and y^f are allocations of each good to each country
- $\alpha > 1/2$ captures home bias

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- $x^h, x^f, y^h,$ and y^f are allocations of each good to each country
- $\alpha > 1/2$ captures home bias
- Endowments are **co-integrated**, and feature **long-run** and **volatility** risks:

$$\Delta \log X_t = \mu_x + z_{1,t-1} - \tau \log(X_{t-1}/Y_{t-1}) + e^{\sigma_{x,t}/2} \sigma \varepsilon_{x,t}$$

$$\Delta \log Y_t = \mu_y + z_{2,t-1} + \tau \log(X_{t-1}/Y_{t-1}) + e^{\sigma_{y,t}/2} \sigma \varepsilon_{y,t}$$

$$z_{j,t} = \rho z_{j,t-1} + \sigma_z \varepsilon_{j,t}, \forall j \in \{1, 2\}$$

- Focus on short-run volatilities of endowments, as in the data.
 - Can extend to accommodate long-run volatility risks

Equilibrium Allocations and Relative Size

- Under complete markets, compute efficient allocations by solving Pareto problem with time-varying weights
- Optimal allocations depend on ratio of Pareto weights (country size) S_t :

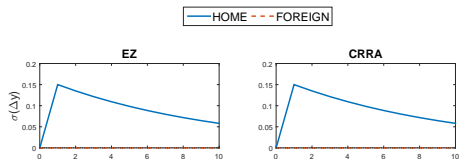
$$S_t = S_{t-1} \cdot \frac{M_t^h}{M_t^f} \cdot \left(\frac{C_t^h / C_{t-1}^h}{C_t^f / C_{t-1}^f} \right), \quad \forall t \geq 1$$

- Evolution of S_t depends on pricing kernels M^h and M^f
- **Under recursive preferences, volatility news are priced**, and affect consumption allocations [▶ Details](#)

Model Calibration

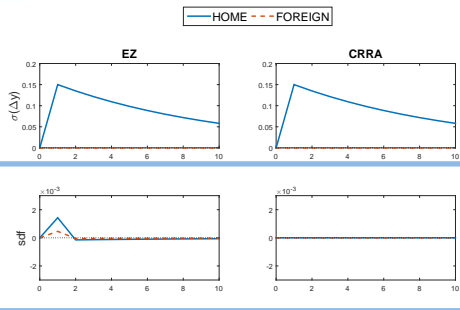
- Calibration for level shocks: similar to Colacito Croce (JPE 2011, JF 2013)
 - Risk aversion is 7
 - Intertemporal elasticity of substitution is 1.5
- Calibration for vol shocks: median estimates in our data
 - Output volatility shocks are persistent
 - Negatively correlated with endowment shocks (-0.12, as in the data)
 - Weakly correlated across countries (0.30)
- Same 'successes' of Colacito Croce (2013) + explains VPTI and vol disconnect

Risk Sharing



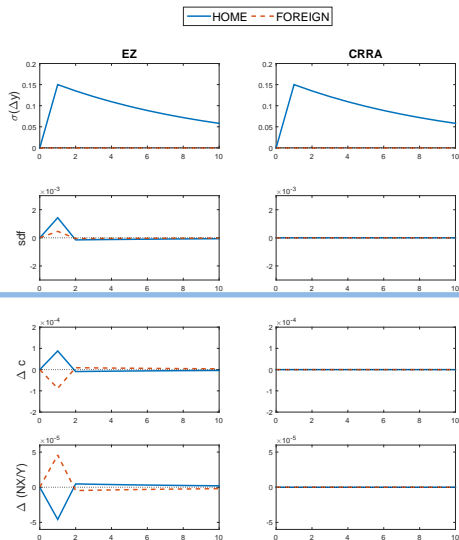
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Risk Sharing



- Home country receives vol shock
- Under EZ utility, vol shock is bad news
 - Home SDF $\uparrow\uparrow$

Risk Sharing



- Home country receives vol shock
- Under EZ utility, vol shock is bad news
 - Home SDF $\uparrow\uparrow$
- Under EZ utility, high vol country receives resources from abroad
 - Home Consumption \uparrow
 - Home NX \downarrow

Unconditional co-movements of volatilities

| | Avg. | Quintiles [1 st ; 4 th] | Bench- mark | No TVV ($\sigma_\sigma = 0$) | CRRA ($\gamma = 7$) |
|--|------|--|----------------|-----------------------------------|--------------------------|
| $corr(\sigma_t(\Delta c_{t+1}), \sigma_t(\Delta y_{t+1}))$ | 0.65 | [0.26; 0.80] | 0.88 | - | 0.98 |
| $corr(\sigma_t(\Delta c_{t+1}), \sigma_t(\Delta c_{t+1}^*))$ | 0.45 | [0.35; 0.66] | 0.35 | -0.93 | 0.50 |

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- CRRA overshoots with both correlations
- Time-varying vol (TVV) brings model with EZ preferences closer to the data

Pass-through and size

| | SWC | US vol shock | Foreign vol shock |
|-------------------------|-------------|--------------|-------------------|
| <i>US/G7 Countries:</i> | | | |
| Data | [0.44 0.51] | [0.43 0.54] | [0.51 0.63] |
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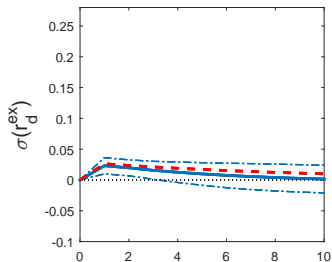
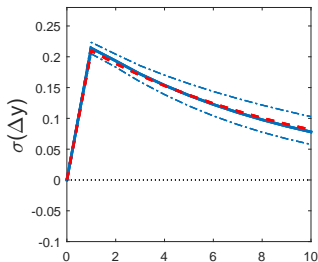
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2 US vs bottom G17 countries

- US has a much larger SWC
- US unloads less vol to smaller countries
- smaller countries unload a lot of vol risk to US

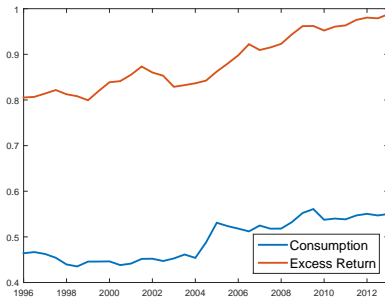
Return Vol Pass-through

$$\text{Pass-through Index} := 1 - \frac{\partial(\sigma_t(r_{d,i}^{\text{ex}}) - \sigma_t(r_{d,US}^{\text{ex}}))}{\partial(\sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US}))}$$



- Excess return pass-through similar to the data (0.89)

Change in Pass-through



| | Benchmark | CRRA |
|------------------------------|-----------|------|
| Consumption vol pass-through | 0.40 | 0.20 |
| Financial pass-through | 0.57 | 0.00 |

FX and Consumption Disconnect in the Model

$$\Delta e_{t+1} \quad (\Delta c_{t+1}^f - \Delta c_{t+1}^h)$$

FX and Consumption Disconnect in the Model

CRRRA

$$\Delta e_{t+1} = (\Delta c_{t+1}^f - \Delta c_{t+1}^h)$$

FX and Consumption Disconnect in the Model

EZ

$$\Delta e_{t+1} = (\Delta c_{t+1}^f - \Delta c_{t+1}^h) - \Delta S_{t+1}$$

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|---|-----------|--|----------------|-----------------------------------|--------------------------|
| | Avg. | Quintiles [1 st ; 4 th] | Bench- mark | No TVV ($\sigma_\sigma = 0$) | CRRA ($\gamma = 7$) |
| <i>Levels Disconnect</i> | | | | | |
| $corr(\Delta cd_{t+1}, \Delta e_{t+1})$ | -0.13 | [-0.19; -0.04] | -0.25 | -0.27 | 1.00 |
| $corr(\Delta \widehat{cd}_{t+4}, \Delta \widehat{e}_{t+4})$ | -0.14 | [-0.29; -0.05] | -0.21 | -0.24 | 1.00 |

- good long-run risks and volatility shocks decrease relative consumption and size of country
- Produces weak negative correlation between the levels of FX and consumption differential, as in the data

FX and Consumption Disconnect in the Model

EZ

$$\Delta e_{t+1} = (\Delta c_{t+1}^f - \Delta c_{t+1}^h) - \Delta S_{t+1}$$

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| $corr(\Delta \widehat{cd}_{t+4}, \Delta \widehat{e}_{t+4})$ | -0.14 | [-0.29; -0.05] | -0.21 | -0.24 | 1.00 |
| <i>Volatility Disconnect</i> | | | | | |
| $corr(\sigma_t(\Delta cd_{t+1}), \sigma_t(\Delta e_{t+1}))$ | 0.20 | [-0.01 0.42] | 0.56 | -0.75 | 1.00 |
| $corr(\sigma_t(\Delta \widehat{cd}_{t+4}), \sigma_t(\Delta \widehat{e}_{t+4}))$ | 0.26 | [-0.02 0.52] | 0.47 | -0.75 | 1.00 |

- CRRA and model with no TVV cannot match this moment
- Volatilities of consumption differential and consumption share:
 - Move in the same direction in response to volatility shocks
 - Move in the opposite direction in response to long-run shocks

Conclusions

1. Domestic volatility risks are "passed through" internationally
2. Volatility pass-through is significant
 - Smaller countries better share volatility risks
3. FX-Vol Disconnect Puzzle
4. Resolve these puzzles with recursive risk sharing of vol shocks

Table 1: Data Summary Statistics

| | G7 Avg. | G17 Avg. | | G17 Quintile | |
|--|---------|----------|----------|-----------------|-----------------|
| | Simple | Simple | Weighted | 1 st | 4 th |
| <i>Consumption growth</i> | | | | | |
| Mean | 1.91 | 1.63 | 1.89 | 1.26 | 2.02 |
| Std. Dev. | 1.75 | 1.99 | 1.67 | 1.34 | 2.47 |
| AR(1) | 0.11 | 0.07 | 0.17 | -0.16 | 0.31 |
| <i>Output growth</i> | | | | | |
| Mean | 1.94 | 1.71 | 1.93 | 1.43 | 2.00 |
| Std. Dev. | 2.21 | 2.97 | 2.02 | 2.01 | 4.43 |
| AR(1) | 0.00 | -0.09 | 0.07 | -0.26 | 0.09 |
| <i>ΔNet Exports over Output:</i> | | | | | |
| Mean | 0.03 | 0.08 | 0.04 | -0.30 | 0.34 |
| Std. Dev. | 1.60 | 2.48 | 1.45 | 1.79 | 3.24 |
| AR(1) | 0.00 | -0.09 | 0.07 | -0.26 | 0.09 |
| <i>Within-Country Correlations:</i> | | | | | |
| Consump. and output growth | 0.67 | 0.51 | 0.71 | 0.35 | 0.72 |
| Consump. and output vol | 0.54 | 0.47 | 0.65 | 0.26 | 0.80 |
| <i>Across-Country Correlations:</i> | | | | | |
| Consump. growth | 0.27 | 0.24 | 0.25 | 0.13 | 0.33 |
| Output growth | 0.15 | 0.14 | 0.14 | 0.06 | 0.20 |
| Consump. vol | 0.51 | 0.47 | 0.45 | 0.35 | 0.66 |
| Output vol | 0.32 | 0.30 | 0.30 | 0.18 | 0.45 |

Table 2: Volatility Risk Pass-Through

| Panel A: Contemporaneous adjustments to relative volatility shocks | | | | | |
|---|----------------|----------------------|-----------------|----------------|--------------|
| $\sigma(\Delta y)$ | Δy | $\sigma(\Delta c)$ | Δc | $\Delta(NX/Y)$ | Pass-through |
| <i>US/G7 Countries:</i> | | | | | |
| 0.21 | -0.46 | 0.10 | -0.20 | -0.25 | 0.52 |
| [0.20 0.22] | [0.09 0.11] | [-0.44 0.03] | [-0.44 0.03] | [-0.49 -0.02] | [0.48 0.56] |
| <i>US/Bottom-10 G17 Countries:</i> | | | | | |
| 0.21 | -0.57 | 0.08 | -0.16 | -0.39 | 0.61 |
| [0.21; 0.22] | [-0.95; -0.19] | [0.07; 0.09] | [-0.41; 0.09] | [-0.73; -0.06] | [0.56; 0.65] |
| Panel B: Pass-through and size | | | | | |
| | | Origin of Vol Shock: | | | |
| | | U.S. | Foreign Country | | |
| <i>US/G7 Countries:</i> | | 0.49 | 0.57 | | |
| | | [0.43; 0.54] | [0.51; 0.63] | | |
| <i>US/Bottom-10 G17 Countries:</i> | | 0.51 | 0.72 | | |
| | | [0.45; 0.57] | [0.66; 0.78] | | |

Table 3: Volatility Disconnect Puzzle

| | G7 Avg. | G17 Avg. | | G17 Quintile | |
|---|---------|----------|----------|-----------------|-----------------|
| | Simple | Simple | Weighted | 1 st | 4 th |
| <i>Levels Disconnect</i> | | | | | |
| $corr(\Delta cd_{t+1}, \Delta e_{t+1})$ | -0.14 | -0.11 | -0.13 | -0.19 | -0.04 |
| $corr(\Delta \widehat{cd}_{t+4}, \Delta \widehat{e}_{t+4})$ | -0.14 | -0.17 | -0.14 | -0.29 | -0.05 |
| <i>Volatility Disconnect</i> | | | | | |
| $corr(\sigma_t(\Delta cd_{t+1}), \sigma_t(\Delta e_{t+1}))$ | 0.20 | 0.21 | 0.20 | -0.01 | 0.42 |
| $corr(\sigma_t(\Delta \widehat{cd}_{t+4}), \sigma_t(\Delta \widehat{e}_{t+4}))$ | 0.27 | 0.25 | 0.26 | -0.02 | 0.52 |

Table 4: Calibration

| Description | Parameter | Value |
|--|--------------------------|------------------------|
| Panel A: Standard Parameters | | |
| Relative Risk Aversion | γ | 7 |
| Intertemporal Elasticity of Substitution | ψ | 1.50 |
| Subjective Discount Factor | δ^4 | 0.98 |
| Degree of Home Bias | α | 0.96 |
| Mean of Endowment Growth | $\mu \cdot 4$ | 2.00% |
| Short-Run Risk Volatility | $\sigma \cdot \sqrt{4}$ | 1.87% |
| Long-Run Risk Autocorrelation | ρ^4 | 0.953 |
| Relative Long-Run Risk Volatility | σ_z / σ | 6.90% |
| Cross-Correlation of Short-Run Shocks | ρ_X | 00.15 |
| Cross-Correlation of Long-Run Shocks | ρ_z | 00.92 |
| Panel B: Time-Varying Short-Run Risk | | |
| Persistence of Short-Run Volatility | ρ_σ | 0.90 [0.89–0.93] |
| Volatility of Short-Run Volatility | σ_{sr} | 0.15 [0.15–0.16] |
| Cross-Correlation of Short-Run Volatility | ρ_{σ,σ^*} | 0.30 [0.13–0.45] |
| Short-Run Volatility Correlation with Short-Run Shocks | $\rho_{\sigma,\Delta y}$ | -0.12 [-0.15 -0.05] |

Table A1: Robustness of Pass-Through Results

| Panel A: Contemporaneous adjustments to relative volatility shocks | | | | | |
|---|----------------|--------------------|----------------------|-----------------|--------------|
| $\sigma(\Delta y)$ | Δy | $\sigma(\Delta c)$ | Δc | $\Delta(NX/Y)$ | Pass-through |
| <i>Global Benchmark, G17 Countries:</i> | | | | | |
| 0.16 | -0.44 | 0.06 | -0.06 | -0.37 | 0.61 |
| [0.15; 0.16] | [-0.67; -0.21] | [0.06; 0.07] | [-0.20; 0.09] | [-0.56; -0.18] | [0.57; 0.64] |
| <i>US/Pooled G7:</i> | | | | | |
| 0.19 | -0.52 | 0.09 | -0.26 | -0.26 | 0.53 |
| [0.19; 0.20] | [-0.83; -0.23] | [0.08; 0.10] | [-0.50; -0.02] | [-0.49; -0.03] | [0.49; 0.56] |
| <i>VAR(2) Model:</i> | | | | | |
| 0.21 | -0.41 | 0.09 | -0.11 | -0.29 | 0.59 |
| [0.20; 0.21] | [-0.71; -0.11] | [0.08; 0.09] | [-0.34; 0.13] | [-0.53; -0.06] | [0.55; 0.62] |
| Panel B: Pass-through and size | | | | | |
| | | | Origin of Vol Shock: | | |
| | | | US | Foreign Country | |
| <i>Global Benchmark/G17 Countries:</i> | | | 0.52 | 0.62 | |
| | | | [0.45; 0.59] | [0.58; 0.66] | |
| <i>US/Pooled G7:</i> | | | 0.47 | 0.64 | |
| | | | [0.43; 0.52] | [0.58; 0.70] | |
| <i>VAR(2):</i> | | | 0.55 | 0.63 | |
| | | | [0.50; 0.60] | [0.58; 0.68] | |

Table B1: Standard Unconditional Moments

| | G-17 Data | | Model | | |
|---|-----------|--|----------------|-----------------------------------|--------------------------|
| | Avg. | Quintiles [1 st , 4 th] | Bench- mark | No TVV ($\sigma_\sigma = 0$) | CRRA ($\gamma = 7$) |
| $corr(\Delta c, \Delta c^*)$ | 0.25 | [0.13; 0.33] | 0.38 | 0.37 | 0.74 |
| $\sigma(\Delta c)(\%)$ | 1.67 | [1.34; 2.47] | 1.85 | 1.82 | 1.64 |
| $\sigma(\Delta c)/\sigma(\Delta y)$ | 0.88 | [0.57; 0.82] | 0.93 | 0.94 | 0.83 |
| $ACF1(\Delta c)$ | 0.17 | [-0.16; 0.31] | 0.06 | 0.07 | 0.08 |
| $\sigma(M)/E(M)(\%)$ | – | – | 47.86 | 47.85 | 11.49 |
| $\sigma(\Delta e)(\%)$ | 10.50 | [10.2; 11.4] | 12.80 | 12.65 | 8.31 |
| $E(r^f)(\%)$ | 1.35 | [1.44; 2.41] | 2.17 | 2.19 | 14.91 |
| $\sigma(r^f)(\%)$ | 1.79 | [1.61; 2.27] | 0.33 | 0.33 | 3.47 |
| $corr(r^f, r^{f*})$ | 0.51 | [0.37; 0.56] | 0.91 | 0.92 | 0.98 |
| $\sigma(\Delta(NX/Y))/\sigma(\Delta y)$ | 0.70 | [0.67; 0.97] | 0.32 | 0.32 | 0.16 |

Volatility Pass-Through Index

[▶ Back](#)

- Using the VAR on

$$\tilde{Y}_{i,t} = \begin{bmatrix} \sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US}) \\ \Delta y_i - \Delta y_{US} \\ \sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}) \\ \Delta c_i - \Delta c_{US} \\ \Delta(NX/Y)_i - \Delta(NX/Y)_{US} \end{bmatrix},$$

the VPTI is

$$VPTI = 1 - \frac{\tilde{\Sigma}_{3,1}}{\tilde{\Sigma}_{1,1}}$$

Volatility Pass-Through Index (cont'd) [▶ Back](#)

- Using the VAR on

$$\tilde{Y}_{i,t} = \left[\underbrace{\sigma_t(\Delta y_{US})}_1, \underbrace{\sigma_t(\Delta y_i)}_2, \underbrace{\Delta y_{US}}_3, \underbrace{\Delta y_i}_4, \underbrace{\sigma_t(\Delta c_{US})}_5, \underbrace{\sigma_t(\Delta c_i)}_6 \right],$$

- VPTI from country i to US

$$VPTI = 1 - \frac{\tilde{\Sigma}_{6,2} - \tilde{\Sigma}_{5,2}}{\Sigma_{2,2}}$$

- VPTI from US to country i

$$VPTI = 1 - \frac{\tilde{\Sigma}_{5,2} - \tilde{\Sigma}_{6,2}}{\Sigma_{1,1}}$$

Volatility shocks are priced ▶ Back

- Consider the case of $\psi = 1$, then

$$U_t = (1 - \delta) \log C_t + \delta \theta \log E_t \exp \left\{ \frac{U_{t+1}}{\theta} \right\}, \quad \theta = 1/(1 - \gamma) < 0$$

- A second order Taylor expansion about $E_t[U_{t+1}]$ yields

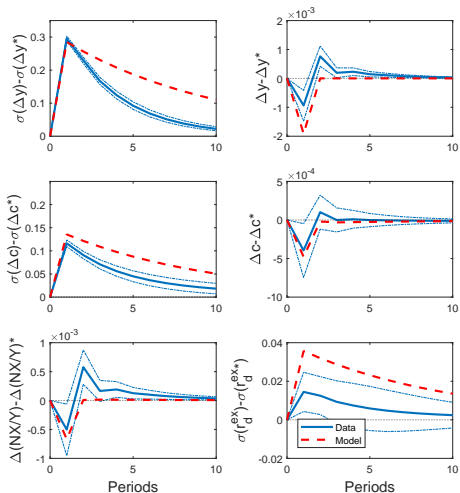
$$U_t \approx (1 - \delta) \log C_t + \delta E_t[U_{t+1}] + \frac{\delta}{2\theta} \text{Var}_t[U_{t+1}]$$

- The SDF is

$$m_t - E_{t-1}[m_t] = -(\Delta c_t - E_{t-1}[\Delta c_t]) + \frac{U_t}{\theta}$$

- If $\text{Var}_t[U_{t+1}] \uparrow$ then $U_t \downarrow$ and $m_t \uparrow$

IRF with correlated level and vol



Pass-through comparison

- With orthogonal shocks

| | US shock | Foreign shock |
|-----------------|--------------|---------------|
| G7 | [0.43, 0.54] | [0.51, 0.63] |
| US vs bottom 10 | [0.45, 0.57] | [0.66, 0.78] |

- With correlated shocks

| | US shock | Foreign shock |
|-----------------|--------------|---------------|
| G7 | [0.50, 0.60] | [0.51, 0.64] |
| US vs bottom 10 | [0.52, 0.63] | [0.78, 0.89] |