Variance Premium, Downside Risk, and Expected Stock Returns

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Motivation and findings

Motivation

- Variance uncertainty
  - Stochastic volatility literature
- Downside risk versus upside potential
  - Roy (1952), Markowitz (1952), etc.
- Variance decomposition
  - Feunou, Jahan-Parvar, and Tédongap (2013), Patton and Sheppard (2015), etc.

Motivation

- Growing literature on variance risk premium (VRP)
  - Carr and Wu (2009), Bollerslev, Tauchen, and Zhou (2009), etc.
- VRP decomposition and substantial improvements of our understanding of equity risk premium time series variation
  - Kilic and Shaliastovich (forthcoming), Feunou, Jahan-Parvar, and Okou (forthcoming), etc.
- How about VRP decomposition and cross-sectional variation in expected returns?

What we do

- Decompose total variance into bad and good components

$$r_{t-1,t} = \sum_{j=1}^{1/\delta} r_{t-1+j\delta} \qquad r_{t-1,t}^2 \neq \sum_{j=1}^{1/\delta} r_{t-1+j\delta}^2 = RV_{t-1,t}$$

$$r_{t-1,t}^2 = r_{t-1,t}^2 \mathbb{I}\left(r_{t-1,t} < 0\right) + r_{t-1,t}^2 \mathbb{I}\left(r_{t-1,t} \geq 0\right)$$

where

$$r_{t-1,t}^2 \mathbb{I}\left(r_{t-1,t} < 0\right) \neq \sum_{j=1}^{1/\delta} r_{t-1+j\delta}^2 \mathbb{I}\left(r_{t-1+j\delta} < 0\right) = RV_{t-1,t}^b$$

$$r_{t-1,t}^2 \mathbb{I}\left(r_{t-1,t} \geq 0\right) \neq \sum_{j=1}^{1/\delta} r_{t-1+j\delta}^2 \mathbb{I}\left(r_{t-1+j\delta} \geq 0\right) = RV_{t-1,t}^g$$

What we do

- Measure premiums associated with variances' fluctuations

$$VRP_t \equiv \mathbb{E}_t^{\mathbb{Q}}\left[r_{t,t+1}^2\right] - \mathbb{E}_t\left[r_{t,t+1}^2\right] = Cov_t\left(Q_{t,t+1}, r_{t,t+1}^2\right)$$

$$VRP_t^b \equiv \mathbb{E}_t^{\mathbb{Q}}\left[r_{t,t+1}^2 \mathbb{I}\left(r_{t,t+1} < 0\right)\right] - \mathbb{E}_t\left[r_{t,t+1}^2 \mathbb{I}\left(r_{t,t+1} < 0\right)\right] = Cov_t\left(Q_{t,t+1}, r_{t,t+1}^2 \mathbb{I}\left(r_{t,t+1} < 0\right)\right)$$

$$VRP_t^g \equiv \mathbb{E}_t\left[r_{t,t+1}^2 \mathbb{I}\left(r_{t,t+1} \geq 0\right)\right] - \mathbb{E}_t^{\mathbb{Q}}\left[r_{t,t+1}^2 \mathbb{I}\left(r_{t,t+1} \geq 0\right)\right] = Cov_t\left(-Q_{t,t+1}, r_{t,t+1}^2 \mathbb{I}\left(r_{t,t+1} \geq 0\right)\right)$$

- Bad VRP ≈ degree to which downside risk may become extreme
- Good VRP ≈ degree to which upside potential may shrink

What we do

- Cost/benefit analysis of total VRP (net effect)

$$VRP_t = VRP_t^b - VRP_t^g$$

- Asymmetry risk premium or ARP (cumulative effect)

$$RA_{t-1,t} \equiv r_{t-1,t}^2 \mathbb{I}\left(r_{t-1,t} \geq 0\right) - r_{t-1,t}^2 \mathbb{I}\left(r_{t-1,t} < 0\right)$$

$$ARP_t \equiv \mathbb{E}_t\left[RA_{t,t+1}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[RA_{t,t+1}\right]$$

$$= VRP_t^b + VRP_t^g$$

What we do

- Analyze the cross-sectional relationship between VRP and expected stock returns
    - Portfolio sorts
    - Cross-sectional regressions
    - Control for systematic (regular and downside) risk
    - Control for other firm characteristics

Main findings

- Bad variance risk premium is important economically: in the cross-section, its one-standard deviation increase is associated with an up to 24% rise in annualized expected excess returns.
- Simultaneously going long stocks with high and short stocks with low bad variance risk premium yields an annualized risk-adjusted expected excess return of 27%.
- Results remain significant in double-sort strategies and cross-sectional regressions controlling for systematic risk and other firm characteristics.

Outline

Data and Measures

Stock data

- Individual daily equity and S&P500 returns from the CRSP database
- Daily risk-free rate, market, size, value, and other pricing factors from Ken French's data library
- Market capitalization and book value from CRSP and Compustat, respectively
- Usual filters applied
- Stock data from January 1994 to December 2015
- Empirical analysis over January 1996 to December 2015

Option data

- Individual equity option prices from the IvyDB OptionMetrics (January 1996 - December 2015)
- Usual filters applied: Carr and Wu (2009), Conrad, Dittmar, and Ghysels (2013), Bakshi, Kapadia, and Madan (2003), Bollerslev, Marrone, Xu, and Zhou (2014)
- Merge option data with stock data following Appendix A.1 in Duarte, Lou, and Sadka (2006).

Measuring risk-neutral expectations

- Use European OTM option contracts and follow Bakshi, Kapadia, and Madan (2003)
- We explicitly prove that

$$\mathbb{E}_t^{\mathbb{Q}}\left[r_{t,t+1}^2 \mathbb{I}\left(r_{t,t+1} < 0\right)\right] \equiv e^{r\tau} V_t^b\left(\tau\right)$$

$$\mathbb{E}_t^{\mathbb{Q}}\left[r_{t,t+1}^2 \mathbb{I}\left(r_{t,t+1} \geq 0\right)\right] \equiv e^{r\tau} V_t^g\left(\tau\right)$$

where

$$V_t^b\left(\tau\right) = \int_0^{S_t} \frac{1 + \ln\left(S_t/K\right)}{K^2/2} P_t\left(\tau; K\right) dK$$

$$V_t^g\left(\tau\right) = \int_{S_t}^{\infty} \frac{1 - \ln\left(K/S_t\right)}{K^2/2} C_t\left(\tau; K\right) dK$$

Measuring real-world expectations

- Assume that

$$r_{t,t+1} \mid \mathcal{I}_t \sim \mathcal{N}\left(\mu_t, \sigma_t^2\right)$$

with

$$\mu_t = \mathbb{E}_t\left[r_{t,t+1}\right] = Z_t^\top \beta_\mu \quad \text{and} \quad \sigma_t^2 = \mathbb{E}_t\left[RV_{t,t+1}\right] = Z_t^\top \beta_\sigma$$

- Therefore

$$\mathbb{E}_t\left[r_{t,t+1}^2\right] = \mu_t^2 + \sigma_t^2$$

$$\mathbb{E}_t\left[r_{t,t+1}^2 \mathbb{I}\left(r_{t,t+1} < 0\right)\right] = \left(\mu_t^2 + \sigma_t^2\right)\Phi\left(-\frac{\mu_t}{\sigma_t}\right) - \mu_t\sigma_t\phi\left(\frac{\mu_t}{\sigma_t}\right)$$

$$\mathbb{E}_t\left[r_{t,t+1}^2 \mathbb{I}\left(r_{t,t+1} > 0\right)\right] = \left(\mu_t^2 + \sigma_t^2\right)\Phi\left(\frac{\mu_t}{\sigma_t}\right) + \mu_t\sigma_t\phi\left(\frac{\mu_t}{\sigma_t}\right)$$

Controls: systematic risk

- Downside risk
  - Generalized Disappointment Aversion (GDA) factor exposures, following Farago and Tédongap (forthcoming)
- Regular risk
  - Market bad VRP
  - Market skewness, following Chang, Christoffersen, and Jacobs (2013)

Controls: other firm characteristics

- Other firm characteristics
  - Relative signed jump variation, following Bollerslev, Zhengzi, and Zhao (2017)
  - Firm skewness, following Conrad, Dittmar, and Ghysels (2013)
  - Idiosyncratic volatility, following Ang, Hodrick, Xing, and Zhang (2006)
  - Illiquidity, following Amihud (2002)

Empirical facts

Where we're headed

- **Portfolio sorts**
- **Fama-MacBeth regressions**

Univariate portfolio sorts: bad VRP, good VRP

	Panel A: Firm Bad VRP						Panel B: Firm Good VRP					
	Quintiles							Quintiles				
	1	2	3	4	5	5-1		1	2	3	4	5	5-1
VRP^b	-181.28	7.89	33.87	70.63	243.75		VRP^g	-67.34	-3.06	15.47	43.25	227.63	
$\mathbb{E}[r]$	-0.52	0.58	1.10	1.40	1.68	2.19		-0.50	0.65	0.96	1.01	1.52	2.02
	(-1.16)	(**2.03**)	(**3.40**)	(**3.12**)	(**2.78**)	(**5.22**)		(-1.15)	(**2.20**)	(**3.28**)	(**2.37**)	(**2.59**)	(**5.76**)
alpha	-0.54	0.57	1.11	1.42	1.71	2.25		-0.51	0.63	0.96	1.01	1.56	2.07
	(-1.26)	(**2.07**)	(**3.34**)	(**3.11**)	(**2.78**)	(**4.95**)		(-1.20)	(**2.08**)	(**3.20**)	(**2.35**)	(**2.69**)	(**5.82**)

Univariate portfolio sorts: total VRP, ARP

Panel C: Firm Total VRP

	Quintiles					
	1	2	3	4	5	5-1
VRP	-341.78	-24.90	14.25	55.28	256.05	
$\mathbb{E}[r]$	0.26	0.74	0.76	1.00	0.91	0.65
	(0.50)	(**2.23**)	(**2.59**)	(**2.70**)	(1.74)	(**2.19**)
alpha	0.27	0.72	0.76	0.99	0.97	0.70
	(0.55)	(**2.23**)	(**2.51**)	(**2.58**)	(1.81)	(**2.20**)

Panel D: Firm ARP

	Quintiles					
	1	2	3	4	5	5-1
ARP	-102.34	22.73	51.39	97.88	321.14	
$\mathbb{E}[r]$	-0.66	0.60	1.19	1.44	2.38	3.04
	(-1.75)	(**2.15**)	(**3.29**)	(**3.25**)	(**3.65**)	(**6.65**)
alpha	-0.71	0.59	1.19	1.46	2.45	3.16
	(**-2.02**)	(**2.11**)	(**3.21**)	(**3.32**)	(**3.67**)	(**6.22**)

Bivariate portfolio sorts: downside risk

Panel A: Market Factor

	Quintiles 1	2	3	4	5	5-1	
1	-0.50	-0.52	-0.11	-0.66	-1.28	-0.78	(-1.06)
2	0.24	0.69	0.66	0.72	0.01	-0.23	(-0.41)
3	0.89	0.95	0.78	0.85	0.70	-0.19	(-0.33)
4	1.21	1.20	1.53	1.30	1.46	0.25	(0.44)
5	1.29	1.35	1.48	1.45	1.92	0.62	(0.83)
5-1	1.79	1.87	1.59	2.11	3.20		
	(3.62)	**(4.75)**	**(3.59)**	**(4.80)**	**(5.15)**		

Panel B: Market Downside Factor

	Quintiles 1	2	3	4	5	5-1	
1	-0.90	-0.11	-0.17	-0.50	-1.13	-0.23	(-0.44)
2	0.55	0.56	0.80	0.42	0.12	-0.43	(-1.22)
3	1.17	0.80	0.91	1.22	1.33	0.16	(0.42)
4	1.64	1.46	1.60	1.10	1.07	-0.58	(-1.16)
5	2.18	1.83	1.49	0.85	0.91	-1.27	(**-2.35**)
5-1	3.08	1.93	1.66	1.35	2.04		
	(5.26)	**(4.16)**	**(3.76)**	**(2.86)**	**(4.11)**		

Panel C: Downstate Factor

	Quintiles 1	2	3	4	5	5-1	
1	-0.92	-0.47	-0.05	-0.39	-1.16	-0.24	(-0.42)
2	0.32	0.67	0.66	0.44	0.27	-0.05	(-0.15)
3	1.13	1.06	0.98	0.94	1.38	0.25	(0.75)
4	1.74	1.90	1.14	1.09	1.23	-0.51	(-0.99)
5	1.70	1.99	1.73	1.20	1.01	-0.69	(-1.45)
5-1	2.62	2.46	1.78	1.59	2.17		
	(4.44)	**(4.74)**	**(3.81)**	**(2.92)**	**(4.07)**		

Panel D: Volatility Factor

	Quintiles 1	2	3	4	5	5-1	
1	-1.09	-0.30	-0.25	0.04	-0.79	0.30	(0.49)
2	0.42	0.52	0.58	0.49	0.43	0.01	(0.03)
3	0.97	1.20	1.05	1.04	0.81	-0.16	(-0.41)
4	1.27	1.19	1.10	1.44	1.67	0.40	(0.86)
5	1.82	1.95	1.43	1.49	1.63	-0.20	(-0.34)
5-1	2.91	2.25	1.67	1.45	2.41		
	(4.95)	**(4.33)**	**(3.90)**	**(3.22)**	**(5.02)**		

Panel E: Volatility Downside Factor

	Quintiles 1	2	3	4	5	5-1	
1	-0.78	0.17	-0.13	-0.53	-1.32	-0.54	(-1.02)
2	0.49	0.79	0.66	0.47	0.25	-0.24	(-0.75)
3	1.02	0.84	1.00	1.13	0.99	-0.03	(-0.10)
4	1.68	1.37	1.33	1.05	1.23	-0.45	(-0.90)
5	1.78	2.08	1.16	1.62	1.42	-0.36	(-0.68)
5-1	2.56	1.91	1.28	2.14	2.74		
	(5.14)	**(3.87)**	**(2.66)**	**(4.01)**	**(4.34)**		

Bivariate portfolio sorts: regular risk

Panel A: Market Bad VRP

	Quintiles 1	2	3	4	5	5-1	
1	-0.99	-0.28	-0.29	-0.19	-0.59	0.40	(0.95)
2	0.26	0.80	0.75	0.43	0.23	-0.04	(-0.10)
3	1.12	1.09	0.86	1.23	1.14	0.02	(0.05)
4	1.46	1.73	1.18	1.29	1.45	-0.01	(-0.02)
5	1.11	1.45	1.90	1.77	1.69	0.58	(1.21)
5-1	2.09	1.73	2.20	1.95	2.28		
	(4.11)	**(3.14)**	**(4.25)**	**(3.80)**	**(4.47)**		

Panel B: Market Risk Neutral Skewness

	Quintiles 1	2	3	4	5	5-1	
1	-0.94	-0.09	-0.14	-0.22	-1.40	-0.46	(-0.83)
2	0.45	0.53	0.47	0.66	0.69	0.24	(0.62)
3	1.25	1.17	0.76	0.95	1.29	0.04	(0.10)
4	1.21	1.57	1.30	1.51	1.05	-0.16	(-0.35)
5	1.59	1.42	1.55	1.93	1.59	-2.2e-3	(-3.7e-3)
5-1	2.53	1.51	1.69	2.16	2.99		
	(4.17)	**(2.93)**	**(3.88)**	**(4.02)**	**(4.89)**		

Bivariate portfolio sorts: other firm characteristics – 1

	Panel A: Illiquidity							Panel B: Idiosyncratic Volatility						
	Quintiles							Quintiles						
	1	2	3	4	5	5-1		1	2	3	4	5	5-1	
1	0.12	-0.26	-0.91	-1.23	-1.96	-2.08	(-**4.33**)	0.51	-0.13	-0.79	-1.47	-2.26	-2.76	(-**4.11**)
2	0.69	0.68	0.42	0.44	0.19	-0.51	(-1.39)	0.69	0.57	0.52	0.05	-0.52	-1.21	(-**2.20**)
3	0.72	1.01	1.04	1.05	0.94	0.21	(0.58)	0.76	0.83	0.84	1.30	0.76	1.3e-3	(2.5e-3)
4	1.20	1.30	1.51	1.43	1.80	0.60	(1.82)	1.19	1.39	1.13	1.31	1.15	-0.04	(-0.08)
5	1.50	1.73	1.36	2.17	1.04	-0.46	(-1.06)	1.40	1.82	1.76	1.47	1.49	0.09	(0.14)
5-1	1.37	1.99	2.26	3.41	2.99			0.90	1.95	2.55	2.94	3.75		
	(**3.82**)	(**5.64**)	(**5.63**)	(**7.45**)	(**7.61**)			(**3.04**)	(**5.31**)	(**5.60**)	(**4.92**)	(**5.27**)		

Bivariate portfolio sorts: other firm characteristics – 2

Panel C: Risk Neutral Skewness

	Quintiles 1	2	3	4	5	5-1	
1	-0.09	-0.08	-0.61	-0.99	-1.19	-1.09	(-**2.55**)
2	0.65	0.77	0.72	0.46	0.34	-0.31	(-1.11)
3	1.05	1.40	1.14	1.07	0.99	-0.06	(-0.21)
4	1.61	1.41	1.68	1.64	1.59	-0.02	(-0.06)
5	1.22	1.82	1.92	2.05	2.02	0.80	(1.75)
5-1	1.31	1.90	2.53	3.04	3.20		
	(**2.64**)	(**3.86**)	(**4.81**)	(**6.59**)	(**6.91**)		

Panel D: Relative Signed Jump Variation

	Quintiles 1	2	3	4	5	5-1	
1	-1.22	-0.64	-0.37	-0.54	-0.09	1.13	(**2.47**)
2	0.18	0.65	0.53	0.43	0.45	0.27	(0.83)
3	1.30	1.05	1.22	1.16	1.13	-0.17	(-0.51)
4	1.51	1.63	1.34	1.08	1.50	-0.01	(-0.02)
5	1.08	1.63	1.53	1.79	1.99	0.91	(1.48)
5-1	2.30	2.27	1.90	2.33	2.08		
	(**4.37**)	(**4.24**)	(**3.47**)	(**4.72**)	(**4.17**)		

Fama-MacBeth regressions: systematic risk

I		II		III		IV		V		VI		VII	
Cst	0.01	Cst	2.7e-3	Cst	0.01	Cst	0.01	Cst	0.01	Cst	0.01	Cst	0.01
	(1.61)		(0.65)		(**2.47**)		(**2.53**)		(**2.71**)		(**3.16**)		(**2.80**)
VRP	0.05	VRP^b	0.47	VRP^b	0.47	VRP^b	0.47	VRP^b	0.50	VRP^b	0.47	VRP^b	0.49
	(1.56)		(**8.18**)		(**8.42**)		(**8.51**)		(**8.93**)		(**8.93**)		(**8.91**)
		VRP^g	0.63	VRP^g	0.65	VRP^g	0.66	VRP^g	0.70	VRP^g	0.69	VRP^g	0.69
			(**9.32**)		(**10.84**)		(**10.90**)		(**11.79**)		(**11.73**)		(**11.55**)
				$\beta_{m,CAPM}$	-3.3e-3	$\beta_{m,SKEW}$	-3.3e-3	$\beta_{m,BTZ}$	-3.3e-3	$\beta_{m,CH}$	-2.6e-3	$\beta_{m,W}$	-3.3e-3
					(-1.07)		(-1.10)		(-1.09)		(-0.99)		(-1.07)
						β_{MSKEW}	0.07	β_{MVRP^b}	5.5e-6	β_{smb}	-2.5e-3	β_X	-4.4e-3
							(1.01)		(0.27)		(-**2.20**)		(-**2.09**)
								β_{MVRP^g}	-1.2e-5	β_{hml}	-2.2e-4	$\beta_{\boldsymbol{D}}$	0.22
									(-1.06)		(-0.16)		(**2.34**)
								β_{VIX}	5.9e-6	β_{mom}	1.3e-3	$\beta_{W\boldsymbol{D}}$	7.7e-6
									(0.17)		(0.55)		(1.28)
												$\beta_{X\boldsymbol{D}}$	9.2e-6
													(1.71)
Adj. R^2	0.77		2.33		5.88		6.24		7.40		9.75		7.32

Fama-MacBeth regressions: other firm characteristics

I		II		VIII		IX	
Cst	7.1e-3	Cst	2.7e-3	Cst	2.9e-	Cst	-5.4e-3
	(1.61)		(0.65)		(0.69)		(-0.24)
VRP	0.05	VRP^b	0.47	VRP^b	0.47	VRP^b	0.64
	(1.56)		(**8.18**)		(**8.26**)		(**4.05**)
		VRP^g	0.63	VRP^g	0.63	VRP^g	1.11
			(**9.32**)		(**9.31**)		(**5.56**)
				RSJ	-6.5e-3	RSJ	5.6e-3
					(-1.81)		(1.51)
						$IVOL$	-0.38
							(-**2.74**)
						Others	...
							(...)
Adj. R^2	0.77	Adj. R^2	2.33	Adj. R^2	3.27	Adj. R^2	14.59

Summary and conclusion

Summary and conclusion

- Large literature on the relationship between VRP and expected returns
- Mostly time series predictability analysis
- We focus on cross-sectional tests using a large cross-section of individual stocks
- We decompose the total VRP into bad and good components that capture insurance costs and compensation benefits for fluctuations in the realized variation in negative and positive stock returns, respectively
- We find strong evidence of a positive cross-sectional relationship between individual firm VRP components and expected returns
- Interesting empirical extension would be to look at the cross-section of other assets such as currencies and commodities

References

Amihud, Y., 2002. Illiquidity and Stock Returns: Cross-section and Time-series Effects. Journal of Financial Markets 5, 31–56.

Ang, A., R. J. Hodrick, Y. Xing, X. Zhang, 2006. The Cross Section of Volatility and Expected Returns. Journal of Finance 11(1), 259–299.

Bakshi, G., N. Kapadia, D. Madan, 2003. Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options. Review of Financial Studies 16(1), 101 – 143.

Bollerslev, T., J. Marrone, L. Xu, H. Zhou, 2014. Stock Return Predictability and Variance Risk Premia, Statistical Inference and International Evidence. Journal of Financial and Quantitative Analysis 3(49), 633–661.

Bollerslev, T., G. Tauchen, H. Zhou, 2009. Expected Stock Returns and Variance Risk Premia. Review of Financial Studies 22(11), 4463–4492.

Bollerslev, T., L. S. Zhengzi, B. Zhao, 2017. Good Volatility, Bad Volatility, and the Cross-Section of Stock Returns. Working Paper.

Carr, P., L. Wu, 2009. Variance Risk Premiums. Review of Financial Studies 22(3), 1311–1341.

Chang, B. Y., P. Christoffersen, K. Jacobs, 2013. Market skewness risk and the cross section of stock returns. Journal of Financial Economics 107(1), 46 – 68.

Conrad, J., R. F. Dittmar, E. Ghysels, 2013. Ex Ante Skewness and Expected Stock Returns. Journal of Finance 68 (1), 85–124.

Duarte, J., X. Lou, R. Sadka, 2006. Can Liquidity Events Explain the Low-Short-Interest Puzzle? Implications from the Options Markets. Working Paper.

Farago, A., R. Tédongap, forthcoming. Downside Risks and the Cross-Section of Asset Returns. Journal of Financial Economics.

Feunou, B., M. Jahan-Parvar, C. Okou, forthcoming. Downside Variance Risk Premium. Journal of Financial Econometrics.

Feunou, B., M. R. Jahan-Parvar, R. Tédongap, 2013. Modeling Market Downside Volatility. Review of Finance 17(1), 443–481.

Kilic, M., I. Shaliastovich, forthcoming. Good and Bad Variance Premia and Expected Returns. Management Science.

Markowitz, H., 1952. Portfolio Selection. Journal of Finance 7(1), 77–91.

Patton, A., K. Sheppard, 2015. Good Volatility, Bad Volatility: Signed Jumps and the Persistence of Volatility. Review of Economics and Statistics 97(3), 683–697.

Roy, A. D., 1952. Safety first and the holding of assets. Econometrica 20, 431–449.