

Is the Distribution of Stock Returns Predictable?

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Predictability of return distribution

Investors are generally concerned with the full return distribution

- Skew, kurtosis preferences
 - ▶ Harvey and Siddique (2000), Dittmar (2002), Guidolin and Timmermann (2008)
- Little is known about predictability of distribution of stock returns
- Is the probability of encountering a significant drop in stock prices time-varying?
- Are periods with surges in market prices predictable and linked to particular states of the economy?
- Answers to these questions have important portfolio implications and help improve our understanding of the economic sources of return predictability

Predictability of return distribution

Use parametric model for return distribution (e.g., EGARCH)

- Danger of using misspecified model

Alternative approach: model return quantiles

- flexibility to study individual return quantiles
- improved approximation as more quantiles are added
- robustness to outliers

Dynamic Quantile Models

- Conditional α -quantile of stock returns, $q_\alpha(r_{t+1}|\mathcal{F}_t)$, uses the Conditional Autoregressive Value-at-Risk (CAViaR) model of Engle and Manganelli (2004):

Dynamic Quantile Model

$$q_\alpha(r_{t+1}|\mathcal{F}_t) = \beta_{0,\alpha} + \beta_{1,\alpha}x_t + \beta_{2,\alpha}q_\alpha(r_t|\mathcal{F}_{t-1}) + \beta_{3,\alpha}|r_t|$$

Estimation

- We adopt the tick-exponential quasi-maximum likelihood estimation approach of Komunjer (2005).
- Estimates of the parameters $\theta_\alpha = (\beta_{0,\alpha}, \beta_{1,\alpha}, \beta_{2,\alpha}, \beta_{3,\alpha})$ solve the objective

The Tick-Exponential QMLE

$$\hat{\theta}_\alpha = \arg \max_{\theta_\alpha} \left\{ T^{-1} \sum_{t=1}^T \ln \varphi_t^\alpha(r_t, q_\alpha(r_t | \mathcal{F}_{t-1}, \theta_\alpha)) \right\}$$

where φ_t^α is a probability density from the tick-exponential family:

$$\varphi_t^\alpha(r_t, q_\alpha) = \exp\left(-\frac{1}{\alpha}(q_\alpha - r_t)\mathbf{1}\{r_t \leq q_\alpha\} + \frac{1}{1-\alpha}(q_\alpha - r_t)\mathbf{1}\{r_t > q_\alpha\}\right)$$

Data

- Sixteen predictor variables analyzed in Goyal and Welch (2006)
- Sample periods vary from 1871-2005 to 1937-2002
- Stock returns are captured by the S&P500 index
- Short T-bill rate is subtracted so we are modeling excess returns

Predictability of Mean Returns

Predictor Variable	OLS Estimate	EGARCH Estimate	
		Mean Equation	Variance Equation
Dividend Price Ratio	0.0000	-0.0021	-0.0057
Dividend Yield	0.0017	-0.0008	-0.0070
Earnings Price Ratio	0.0051	0.0024	-0.0053
Smoothed Earnings Price Ratio	0.1332	0.0550	-0.3043
Book to Market Ratio	0.0106	0.0034	-0.0273
T-bill	-0.0890	-0.1569***	0.1859
Long Term Yield	-0.0604	-0.1058*	0.0250
Term Spread	0.2040	0.1553	-0.9148
Default Yield Spread	0.0648	-0.0914	2.3712
Default Return Spread	0.1436	-0.0802	-7.4058**
Cross Sectional Premium	1.7783**	2.0842***	-1.0271
Long Term Rate of Returns	0.0935	0.1425**	-2.2480
Stock Variance	-0.1533	-0.5251	6.1286
Dividend Payout Ratio	-0.0086	-0.0078**	-0.0037
Net Equity Expansion	-0.2177***	-0.1776**	0.4857
Inflation	-0.4757*	-0.7113***	-1.6514

Predictability of quantiles

Table 3: Coefficient estimates for linear quantile prediction models

(a) Quantiles 0.05, 0.1, 0.3, 0.5

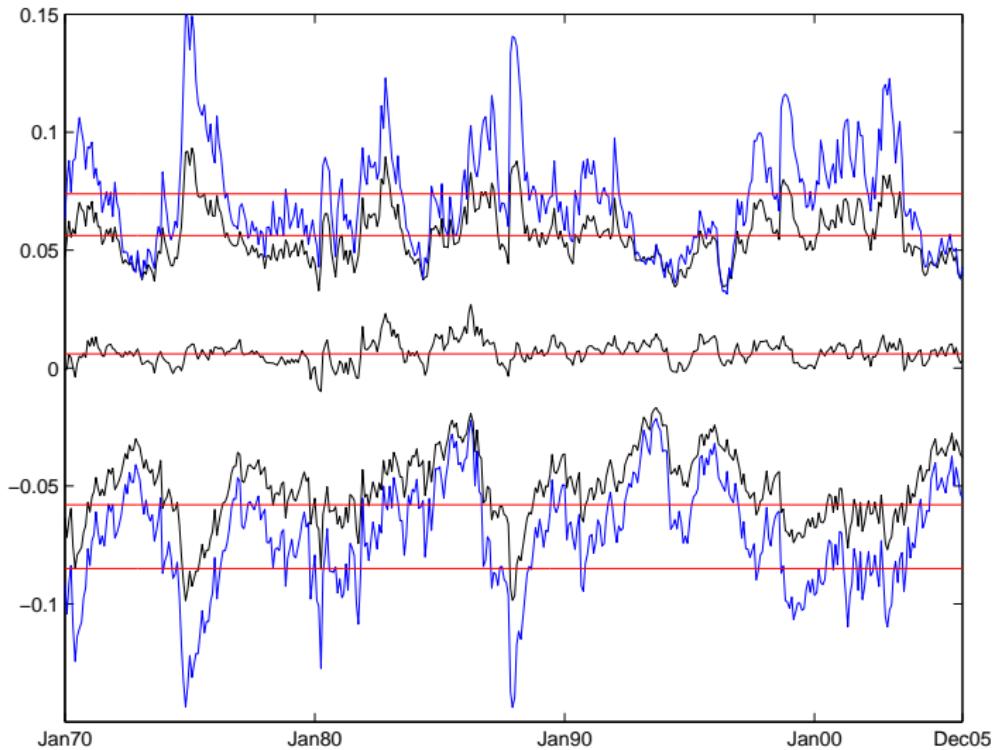
	Quantile			
	0.05	0.1	0.3	0.5
e10/p	-0.3834	-0.1124	0.0143	0.0037
tbl	0.1147	0.0142	-0.1038	-0.1390**
tms	-0.0589	0.1501	0.1257	0.1262
dfy	-4.1446***	-2.9432***	-0.9104**	0.0387
svar	-10.2609***	-4.1415**	-2.2660***	-0.6413
ntis	-0.4655***	-0.4646**	-0.1712*	-0.1108

(b) Quantiles 0.7, 0.9, 0.95

	Quantile			Bonferroni
	0.7	0.9	0.95	
e10/p	0.1611***	0.2811*	0.4193*	0.0330
tbl	-0.1361	-0.1920***	-0.2384***	0.0000
tms	0.0939	0.5073**	0.7515**	0.2530
dfy	0.6163*	2.1080***	3.4645***	0.0000
svar	2.1804**	6.2257***	8.5168***	0.0000
ntis	-0.0676	-0.0165	0.0053	0.0220

Quantile forecasts using long-term rate as predictor

Figure 2: Quantile predictions from the linear autoregressive quantile model



Out-of-sample forecasting experiment

- Out-of-sample period is 1970 - 2006
- Parameter estimates updated recursively each month
- Challenging sample period!

Mean Out-of-Sample Loss

	0.05		0.1		0.5		0.9		0.95	
	DQ	EGARCH	DQ	EGARCH	DQ	EGARCH	DQ	EGARCH	DQ	EGARCH
d/p	5.5325	5.4117	8.6594	8.5691	16.9466	16.9628	7.1562	7.2842	4.3128	4.3689
d/y	5.5377	5.4415	8.6755	8.6225	16.9264	17.0194	7.1469	7.2319	4.2874	4.3315
e/p	5.5265	5.4520	8.6601	8.6068	16.9785	17.0409	7.1392	7.1169	4.2612	4.2583
e10/p	5.5501	5.4903	8.6841	8.6955	17.0794	17.0647	7.1194	7.1809	4.2700	4.2954
b/m	5.8555	5.5734	9.1355	8.7881	17.1374	17.1346	7.2975	7.5020	4.2745	4.4445
tbl	5.4715	5.3675	8.7043	8.6458	16.9280	16.9515	7.2693	7.3130	4.4277	4.4339
lty	5.5751	5.4967	8.8132	8.7324	16.9988	16.9648	7.4253	7.2707	4.4752	4.4826
tms	5.3816	5.2325	8.6087	8.4751	17.0521	17.0161	7.1905	7.3417	4.3665	4.3245
dfy	5.6638	5.4195	8.8022	8.5669	17.0254	16.9396	7.0867	7.2605	4.3031	4.2746
dfr	5.4113	5.2552	8.6974	8.5334	17.0025	16.9772	7.1913	7.3589	4.3750	4.3938
csp	5.6114	5.5068	8.8374	8.7337	17.5161	17.4988	7.3926	7.3155	4.5084	4.4037
ltr	5.4742	5.3150	8.6973	8.5041	17.1170	17.0185	7.2641	7.3258	4.3513	4.3093
svar	5.4263	5.4105	8.5910	8.5580	16.9599	16.9706	7.0681	7.1166	4.3282	4.2730
d/e	5.5314	5.3905	8.6965	8.5337	17.0952	16.9769	7.0684	7.2289	4.3216	4.2919
ntis	5.7265	5.4969	8.8886	8.4958	17.0437	16.9286	7.1779	7.3450	4.3492	4.3333
infl	5.5254	5.4157	8.6535	8.5632	16.7376	16.7958	7.1001	7.1708	4.3418	4.2861
EW	5.4690	5.3898	8.5754	8.5196	16.8877	16.8914	7.0601	7.2077	4.3012	4.3037
EG		5.4167		8.5732		16.9866		7.1142		4.2542
PQ		5.4560		8.6685		16.9318		7.2852		4.4085

Economic Significance

Under power utility the optimal portfolio weights for period t solve

$$w_t^* = \arg \max_{w_t} \int_{-\infty}^{\infty} \frac{\beta}{1 - \gamma} (1 + r_t^f + w_t \rho_{t+1})^{1-\gamma} f(\rho_{t+1} | \mathcal{F}_t) d\rho_{t+1}$$

We assume that the distribution can be approximated by

$$f(\rho_{t+1} | \mathcal{F}_t) = \begin{cases} \frac{1}{\sqrt{2\pi}\tilde{\sigma}_t} \exp(-(\rho_{t+1} - \tilde{\mu}_t)^2 / 2\tilde{\sigma}_t^2), & \text{if } \rho_{t+1} \leq \tilde{q}_{t+1|t}^{0.05}; \\ \frac{1}{20(\tilde{q}_{t+1|t}^{0.10} - \tilde{q}_{t+1|t}^{0.05})}, & \tilde{q}_{t+1|t}^{0.05} < \rho_{t+1} \leq \tilde{q}_{t+1|t}^{0.10}; \\ \frac{1}{10(\tilde{q}_{t+1|t}^{\alpha+0.10} - \tilde{q}_{t+1|t}^{\alpha})}, & \tilde{q}_{t+1|t}^{\alpha} < \rho_{t+1} \leq \tilde{q}_{t+1|t}^{\alpha+0.10} \\ & \text{for } \alpha \in \mathcal{A}; \\ \frac{1}{20(\tilde{q}_{t+1|t}^{0.90} - \tilde{q}_{t+1|t}^{0.95})}, & \tilde{q}_{t+1|t}^{0.90} < \rho_{t+1} \leq \tilde{q}_{t+1|t}^{0.95}; \\ \frac{1}{\sqrt{2\pi}\tilde{\sigma}_t} \exp(-(\rho_{t+1} - \tilde{\mu}_t)^2 / 2\tilde{\sigma}_t^2), & \text{if } \rho_{t+1} > \tilde{q}_{t+1|t}^{0.95}; \end{cases}$$

optimization (cont'd)

$$w_t^* = \arg \max_{w_t}$$

$$\int_{-\infty}^{\tilde{q}_{t+1|t}^{0.05}} \frac{\beta}{1-\gamma} (1 + r_t^f + w_t \rho_{t+1})^{1-\gamma} \frac{1}{\sqrt{2\pi}\tilde{\sigma}_t} \exp(-(\rho_{t+1} - \tilde{\mu}_t)^2/2\tilde{\sigma}_t^2) d\rho_{t+1}$$

$$+ \int_{\tilde{q}_{t+1|t}^{0.05}}^{\tilde{q}_{t+1|t}^{0.10}} \frac{\beta}{1-\gamma} (1 + r_t^f + w_t \rho_{t+1})^{1-\gamma} \frac{1}{20(\tilde{q}_{t+1|t}^{0.10} - \tilde{q}_{t+1|t}^{0.05})} d\rho_{t+1}$$

$$+ \sum_{\alpha \in \mathcal{A}} \int_{\tilde{q}_{t+1|t}^{\alpha}}^{\tilde{q}_{t+1|t}^{\alpha+0.10}} \frac{\beta}{1-\gamma} (1 + r_t^f + w_t \rho_{t+1})^{1-\gamma} \frac{1}{10(\tilde{q}_{t+1|t}^{\alpha+0.10} - \tilde{q}_{t+1|t}^{\alpha})} d\rho_{t+1}$$

$$+ \int_{\tilde{q}_{t+1|t}^{0.90}}^{\tilde{q}_{t+1|t}^{0.95}} \frac{\beta}{1-\gamma} (1 + r_t^f + w_t \rho_{t+1})^{1-\gamma} \frac{1}{20(\tilde{q}_{t+1|t}^{0.95} - \tilde{q}_{t+1|t}^{0.90})} d\rho_{t+1}$$

$$+ \int_{\tilde{q}_{t+1|t}^{0.95}}^{+\infty} \frac{\beta}{1-\gamma} (1 + r_t^f + w_t \rho_{t+1})^{1-\gamma} \frac{1}{\sqrt{2\pi}\tilde{\sigma}_t} \exp(-(\rho_{t+1} - \tilde{\mu}_t)^2/2\tilde{\sigma}_t^2) d\rho_{t+1}$$

Incorporating the analytical solutions to the integrals,

$$w_t^* = \arg \max_{w_t} \int_{-\infty}^{\tilde{q}_{t+1|t}^{0.05}} \frac{\beta}{1-\gamma} (1 + r_t^f + w_t \rho_{t+1})^{1-\gamma} \frac{1}{\sqrt{2\pi}\tilde{\sigma}_t} \exp(-(\rho_{t+1} - \tilde{\mu}_t)^2/2\tilde{\sigma}_t^2) d\rho_{t+1}$$
$$+ \frac{\beta}{(1-\gamma)(2-\gamma)w_t} \left[\frac{1}{20(\tilde{q}_{t+1|t}^{0.10} - \tilde{q}_{t+1|t}^{0.05})} [(1 + r_t^f + w_t \tilde{q}_{t+1|t}^{0.10})^{2-\gamma} - (1 + r_t^f + w_t \tilde{q}_{t+1|t}^{0.05})^{2-\gamma}] \right.$$
$$+ \sum_{\alpha \in \mathcal{A}} \frac{1}{10(\tilde{q}_{t+1|t}^{\alpha+0.10} - \tilde{q}_{t+1|t}^{\alpha})} [(1 + r_t^f + w_t \tilde{q}_{t+1|t}^{\alpha+0.10})^{2-\gamma} - (1 + r_t^f + w_t \tilde{q}_{t+1|t}^{\alpha})^{2-\gamma}]$$
$$\left. + \frac{1}{20(\tilde{q}_{t+1|t}^{0.95} - \tilde{q}_{t+1|t}^{0.90})} [(1 + r_t^f + w_t \tilde{q}_{t+1|t}^{0.95})^{2-\gamma} - (1 + r_t^f + w_t \tilde{q}_{t+1|t}^{0.90})^{2-\gamma}] \right]$$
$$+ \int_{\tilde{q}_{t+1|t}^{0.95}}^{+\infty} \frac{\beta}{1-\gamma} (1 + r_t^f + w_t \rho_{t+1})^{1-\gamma} \frac{1}{\sqrt{2\pi}\tilde{\sigma}_t} \exp(-(\rho_{t+1} - \tilde{\mu}_t)^2/2\tilde{\sigma}_t^2) d\rho_{t+1}$$

Portfolio Allocation Exercise Results

	Certainty Equivalent Return					
	Log Utility		$\gamma = 2.5$		$\gamma = 5.0$	
	DQ	EGARCH	DQ	EGARCH	DQ	EGARCH
d/p	6.36%	6.70%	5.75%	6.07%	5.68%	5.80%
d/y	6.32%	6.34%	5.61%	5.69%	5.75%	5.83%
e/p	6.62%	5.50%	5.55%	5.60%	5.64%	5.79%
e10/p	6.10%	6.20%	5.63%	6.00%	5.73%	5.99%
b/m	5.34%	6.98%	4.42%	5.09%	2.83%	3.30%
tbl	7.82%	8.25%	7.29%	6.98%	6.64%	6.52%
lty	7.53%	7.41%	6.90%	6.70%	6.46%	6.35%
tms	9.48%	9.17%	8.05%	8.20%	6.33%	6.68%
dfy	5.89%	7.05%	4.35%	4.86%	4.83%	5.18%
dfr	5.99%	7.24%	4.74%	4.94%	5.05%	5.15%
csp	6.74%	8.08%	6.52%	7.87%	6.45%	7.09%
ltr	8.35%	8.55%	7.65%	7.28%	6.08%	6.61%
svar	6.29%	6.50%	5.64%	6.08%	5.68%	5.97%
d/e	6.69%	6.64%	4.53%	5.19%	4.24%	5.49%
ntis	6.68%	6.96%	5.27%	5.54%	4.89%	5.16%
infl	7.87%	9.17%	7.28%	8.38%	6.69%	7.35%
EW	8.47%	8.87%	6.92%	7.66%	6.47%	6.85%
EG		6.56%		6.21%		6.09%
PQ		6.52%		6.20%		6.09%

Option Trading Strategies

Compare dynamic quantile forecasts to VIX-implied quantiles:

$$\hat{q}_{\alpha,t}^{\text{VIX}} = \bar{\mu}_t + q_{\alpha,N} \hat{\sigma}_{\text{VIX},t}$$

$q_{\alpha,N}$: α -quantile of the standard normal distribution

Option Trading Strategies

- 1 If $\hat{q}_{t+1|t}^{\alpha} - \hat{q}_{\text{VIX},t+1}^{\alpha} > \overline{\hat{q}_{t+1|t}^{\alpha} - \hat{q}_{\text{VIX},t+1}^{\alpha}}$ for $\alpha = 0.90$ or 0.95 , buy call with highest strike.
- 2 If $\hat{q}_{t+1|t}^{\alpha} - \hat{q}_{\text{VIX},t+1}^{\alpha} < \overline{\hat{q}_{t+1|t}^{\alpha} - \hat{q}_{\text{VIX},t+1}^{\alpha}}$ for $\alpha = 0.90$ or 0.95 , sell call with highest strike.
- 3 If $\hat{q}_{t+1|t}^{\alpha} - \hat{q}_{\text{VIX},t+1}^{\alpha} < \overline{\hat{q}_{t+1|t}^{\alpha} - \hat{q}_{\text{VIX},t+1}^{\alpha}}$ for $\alpha = 0.05$ or 0.10 , buy put with lowest strike.
- 4 If $\hat{q}_{t+1|t}^{\alpha} - \hat{q}_{\text{VIX},t+1}^{\alpha} > \overline{\hat{q}_{t+1|t}^{\alpha} - \hat{q}_{\text{VIX},t+1}^{\alpha}}$ for $\alpha = 0.05$ or 0.10 , sell put with lowest strike.

Stochastic Dominance Tests (Linton et al (2005))

$\bar{F}_{iN}(y)$: empirical cdf,

$$\bar{F}_{iN}(y) = \frac{1}{N} \sum_{j=1}^N 1(y_{ij} \leq y)$$

Test statistic for stochastic dominance, d_s^* , is

$$d_s^* = \max_{y \in \mathcal{Y}} \sqrt{N} [\bar{D}_{DQ}^{(s)}(y) - \bar{D}_{Bmk}^{(s)}(y)]$$

where

$$\bar{D}_i^{(1)}(y) = \bar{F}_{iN}(y)$$

$$\bar{D}_i^{(s)}(y) = \int_{-\infty}^y \bar{D}_i^{(s-1)}(t) dt \quad \text{for } s \geq 2$$

$$\mathbf{H}_0 : d_2^* \leq 0$$

$$\mathbf{H}_1 : d_2^* > 0$$

Option Trading Results Based on VIX Implied Quantiles

	Test Statistic for H_0 : Active Strategy		Test Statistic for H_0 : Benchmark Strategy	
	SSD Benchmark Strategy		SSD Active Strategy	
	EW DQ	EW EGARCH	EW DQ	EW EGARCH
Strategy 1 for $\alpha = 0.90$	0.0000	0.0000	84.5860***	68.1610***
Strategy 1 for $\alpha = 0.95$	0.0000	0.0000	56.3660***	54.4850***
Strategy 2 for $\alpha = 0.90$	12.9520	23.6610	28.0750*	31.6200***
Strategy 2 for $\alpha = 0.95$	23.2270	22.5030	29.3770**	24.5290
Strategy 3 for $\alpha = 0.05$	0.0000	0.0000	78.6530***	80.2440***
Strategy 3 for $\alpha = 0.10$	0.0000	0.0000	84.0790***	80.1720***
Strategy 4 for $\alpha = 0.05$	24.6020	16.7150	18.7410	31.6930
Strategy 4 for $\alpha = 0.10$	30.5350	11.7220	13.3860	36.6130

Conclusion

- Consistent with earlier studies we find little evidence to suggest that the center (mean) of the return distribution can be predicted
- Tails of stock returns can be predicted using state variables from the literature
- Equal-weighted combinations of quantile or EGARCH forecasts do well