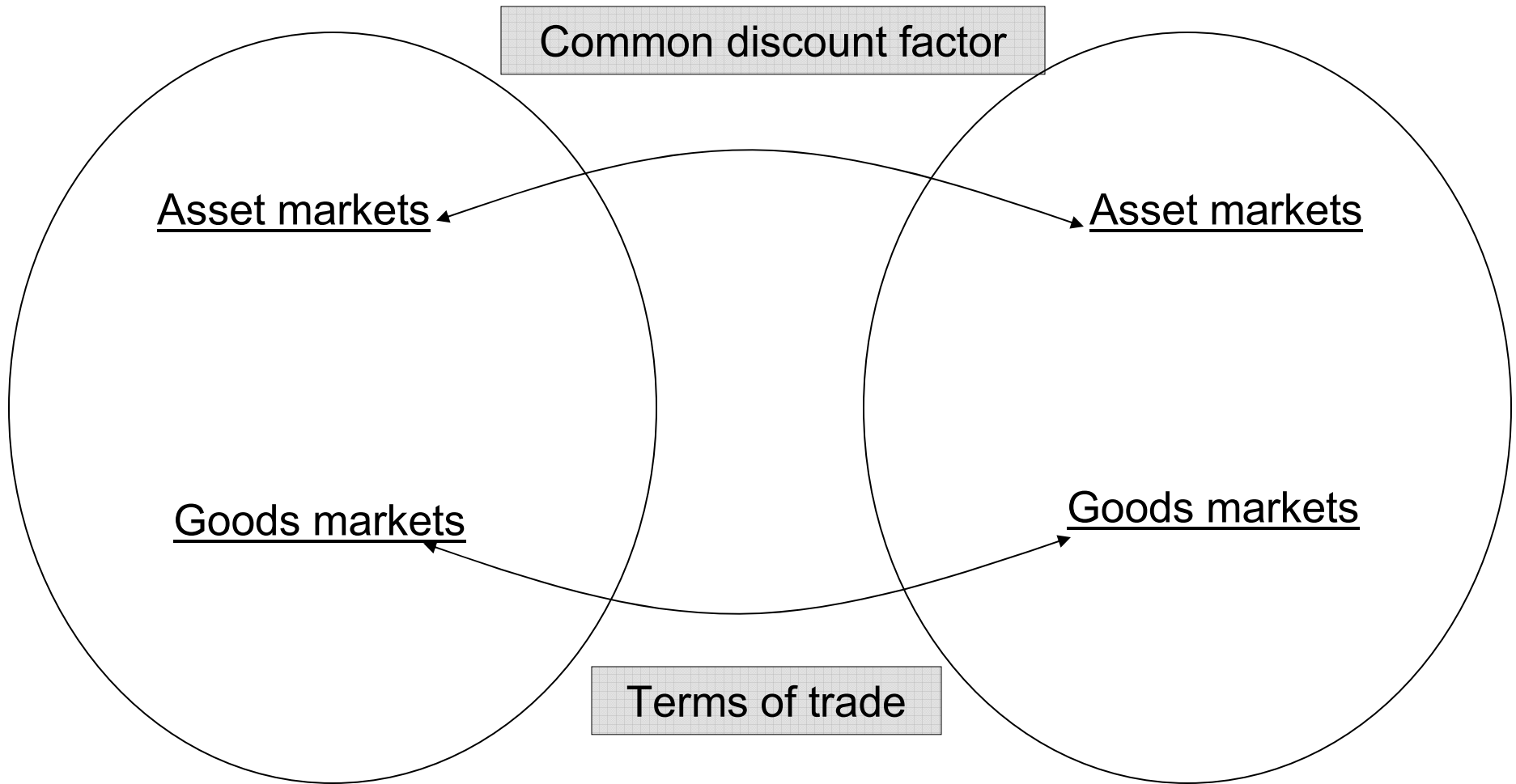# The Role of Portfolio Constraints in the International Propagation of Shocks

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Channels of Comovement

Common discount factor

Asset markets

Asset markets

Goods markets

Goods markets

Terms of trade

Correlations go up during Crises

- Empirical evidence: correlations of stock market and exchange rate returns in tranquil times and during crises:

|	Stock markets		Exchange rates	
	Tranquil times	Crises	Tranquil times	Crises
Emerging economies	27%	47%	7%	20%
Developed economies	37%	43%	55%	58%

Source: authors' calculations. The sample includes the Mexico 1994, Thailand 1997, Hong Kong 1997, Korea 1997-98, Asia 1997-98, Russia 1998, Brazil 1999, Turkey 2001, and Argentina 2002 crises. All correlations represent bilateral correlations of all stated countries with the country in crisis.

Economy
Simple three-country model with three risky stocks
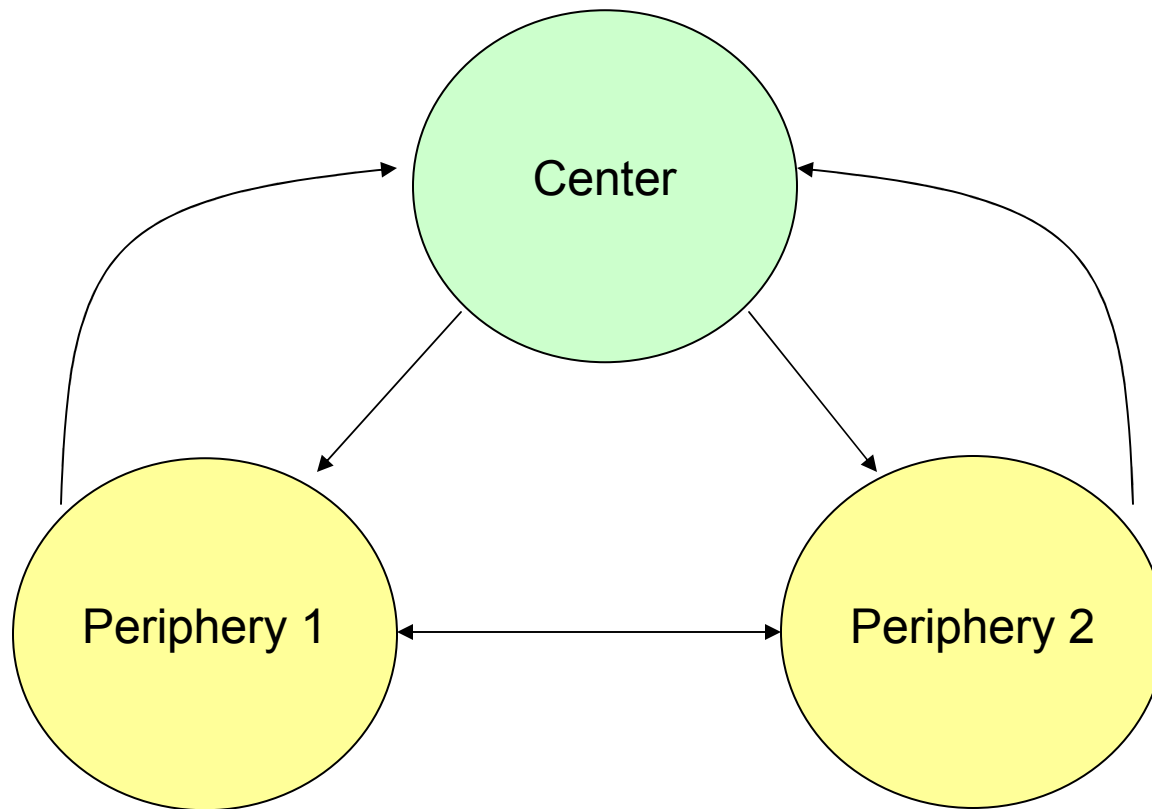
Economy with Portfolio Constraints
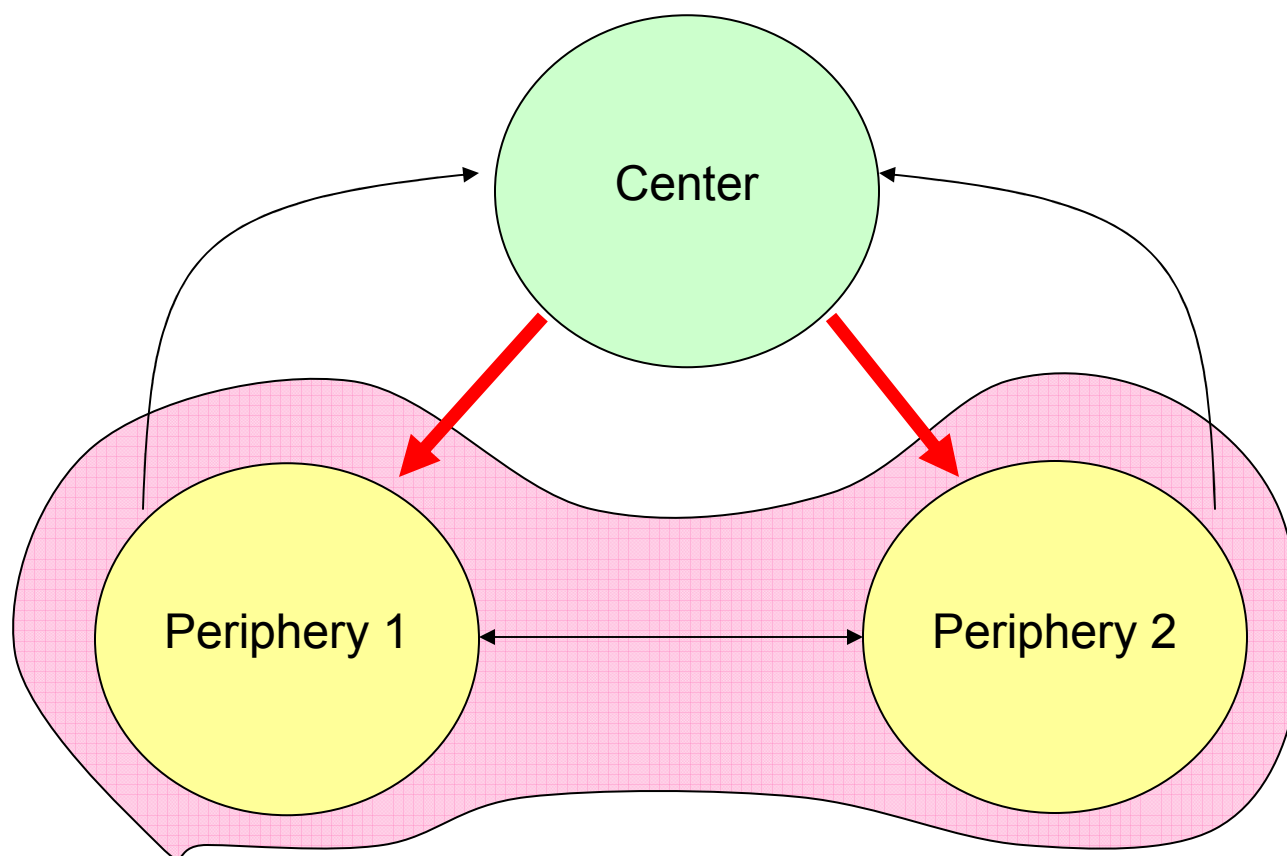
Flow of goods: Unrestricted

Flow of capital: Holdings of Center are restricted

Our Main Contributions

- A tractable multi-asset multi-good model with heterogeneous agents and market frictions
  - Stock prices can be computed in closed form
  - Some simple new economic intuitions

- An explanation for "excess" comovement of the stock markets in the Periphery

Model

- Finite horizon $[0, T]$
- Three countries: Center (big - 0) and
                    two Periphery (smaller – 1,2)
- Information structure generated by Brownian motion $w = \left(w^0, w^1, w^2 \right)$

- Each country produces its own good; Lucas tree with output

$$dY^j(t) = \mu_{Y^j}(t) Y^j(t) dt + \sigma_{Y^j}(t) Y^j(t) dw^j(t)$$

- Prices of goods are p^j
- Terms of trade: $q^1 = p^1 / p^0, \quad q^2 = p^2 / p^0$

Model

- ## Investment opportunities

  - ### One stock in each country S^j: claim to its output Y^j

    - positive net supply

  - ### International bond B: instantaneously riskless in the world numeraire

    - zero net supply

Model

- ## Each consumer-investor *j* maximizes

$$E\left[\int_0^T u_j\left(C_j^0(t),\ C_j^1(t),\ C_j^2(t)\right)dt\right]$$

$$u_j\left(C_j^0,\ C_j^1,\ C_j^2\right)=\alpha_j\log\left(C_j^j\right)+\frac{1-\alpha_j}{2}\sum_{i\neq j}\log\left(C_j^i\right)$$

Subject to standard dynamic budget constraint

- home bias in consumption: $\alpha_j > 1/3$
- we allow $\alpha_1(t)$ and $\alpha_2(t)$ to be stochastic
- "demand shifts" of Dornbusch, Fischer, and Samuelson (1977)

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Benchmark Unconstrained Equilibrium

- It is OK to consider the planner's problem
  - No financial market degeneracy of Helpman and Razin (1978), Cole and Obstfeld (1991), and Zapatero (1995)

- The planner maximizes

$$E\int_{0}^{T}\left[\lambda_0 u_0(\cdot) + \lambda_1 u_1(\cdot) + \lambda_2 u_2(\cdot)\right]dt$$

  - with constant weights λ_j
  - normalize $\lambda_0 = 1$

Benchmark Unconstrained Equilibrium

- Two useful properties of equilibrium:

  – Constant wealth distribution: $\lambda_1 = \dfrac{W_1(t)}{W_0(t)}$ and $\lambda_2 = \dfrac{W_2(t)}{W_0(t)}$

  – All countries hold identical (Merton's mean-variance) portfolios

Benchmark Unconstrained Equilibrium

- We can compute the terms of trade and stock prices in closed form. For example,

$$S^j(t) = E_t \int_t^T \frac{\xi(s)}{\xi(t)} \, p^j(s) Y^j(s) \, ds = p^j(t) \, Y^j(t)(T-t)$$

- Dynamics:

$$
\begin{bmatrix}
\frac{dq^1(t)}{q^1(t)} \\[4pt]
\frac{dq^2(t)}{q^2(t)} \\[4pt]
\frac{dS^0(t)}{S^0(t)} \\[4pt]
\frac{dS^1(t)}{S^1(t)} \\[4pt]
\frac{dS^2(t)}{S^2(t)}
\end{bmatrix}
= I(t)dt +
\underbrace{
\begin{bmatrix}
a(t) & b(t) & 1 & -1 & 0 \\[4pt]
\widetilde{a}(t) & \widetilde{b}(t) & 1 & 0 & -1 \\[4pt]
-X_{\alpha_1}(t) & -X_{\alpha_2}(t) & \beta M(t) & \frac{1-\beta}{2}\frac{M(t)}{q^1(t)} & \frac{1-\beta}{2}\frac{M(t)}{q^2(t)} \\[4pt]
a(t) - X_{\alpha_1}(t) & b(t) - X_{\alpha_2(t)} & \beta M(t) & \frac{1-\beta}{2}\frac{M(t)}{q^1(t)} & \frac{1-\beta}{2}\frac{M(t)}{q^2(t)} \\[4pt]
\widetilde{a}(t) - X_{\alpha_1}(t) & \widetilde{b}(t) - X_{\alpha_2(t)} & \beta M(t) & \frac{1-\beta}{2}\frac{M(t)}{q^1(t)} & \frac{1-\beta}{2}\frac{M(t)}{q^2(t)}
\end{bmatrix}
}_{\Theta_u(t)}
\begin{bmatrix}
d\alpha_1(t) \\[4pt]
d\alpha_2(t) \\[4pt]
\sigma_{Y^0}(t)dw^0(t) \\[4pt]
\sigma_{Y^1}(t)dw^1(t) \\[4pt]
\sigma_{Y^2}(t)dw^2(t)
\end{bmatrix}
$$

Benchmark Unconstrained Equilibrium

Variable/ Effects of	$d\alpha_1(t)$	$d\alpha_2(t)$	$dw^0(t)$	$dw^1(t)$	$dw^2(t)$
$\dfrac{dq^1(t)}{q^1(t)}$	$+$	$-^{A_1}$	$+$	$-$	0
$\dfrac{dq^2(t)}{q^2(t)}$	$-^{A_1}$	$+$	$+$	0	$-$
$\dfrac{dS^0(t)}{S^0(t)}$	$-^{A_2}$	$-^{A_2}$	$+$	$+$	$+$
$\dfrac{dS^1(t)}{S^1(t)}$	$+^{A_1}$	$-^{A_2}$	$+$	$+$	$+$
$\dfrac{dS^2(t)}{S^2(t)}$	$-^{A_2}$	$+^{A_1}$	$+$	$+$	$+$

A_1: Periphery countries are small

A_2: Periphery countries are similar in size

Equilibrium with Portfolio Constraints

- Class of portfolio constraints:

$$x_0(t) \in K(t)$$

  - x_0 is the fraction of wealth invested by the Center in each country
  - $K \in R^3$ is a closed convex set
    - This set may be stochastic; it may depend on
    $$S^j, \ p^j, \ Y^j, \ldots$$

What can we capture?

- Institutionally imposed portfolio constraints:
  - Concentration constraints
  - Collateral constraints
  - VaR's
  - Margin requirements

- Market segmentation

Equilibrium with Portfolio Constraints

- Partial equilibrium: methodology
  - Periphery countries face the investment opportunity set

$$\frac{d\,B(t)}{B(t)} = r(t)\,dt$$

$$\frac{d\,S^j(t)}{S^j(t)} = \mu^j(t)\,dt + \sigma^j(t)\,dw(t)$$

  - Center faces

$$\frac{d\,B(t)}{B(t)} = (r(t) + \delta(v(t)))\,dt$$

$$\frac{d\,S^j(t)}{S^j(t)} = (\mu^j(t) + v^j(t) + \delta(v(t)))\,dt + \sigma^j(t)\,dw(t)$$

Equilibrium with Portfolio Constraints

- Equilibrium: Same problem as before but now the weights λ_j are stochastic (again, normalize $\lambda_0 = 1$)

$$E\int_0^T \left[\lambda_0(t)u_0(\cdot) + \lambda_1(t)u_1(\cdot) + \lambda_2(t)u_2(\cdot)\right]dt$$

- The weights in the "planner's problem" again reflect the wealth distribution

$$\lambda_1(t) = \frac{W_1(t)}{W_0(t)} \qquad \text{and} \qquad \lambda_2(t) = \frac{W_2(t)}{W_0(t)}$$

- But the wealth distribution is now stochastic.

- Still wealth shares of the Periphery countries move in tandem:

$$\frac{d\lambda_1(t)}{\lambda_1(t)} = \frac{d\lambda_2(t)}{\lambda_2(t)} = \frac{d\lambda(t)}{\lambda(t)}$$

Equilibrium with Portfolio Constraints

- **Proposition 2**:
  - The constraint adds one additional factor, λ

$$
\begin{bmatrix}
\frac{dq^1(t)}{q^1(t)} \\
\frac{dq^2(t)}{q^2(t)} \\
\frac{dS^0(t)}{S^0(t)} \\
\frac{dS^1(t)}{S^1(t)} \\
\frac{dS^2(t)}{S^2(t)}
\end{bmatrix}
=
\begin{bmatrix}
A(t) \\
\widetilde{A}(t) \\
-X_\lambda(t) \\
A(t) - X_\lambda(t) \\
\widetilde{A}(t) - X_\lambda(t)
\end{bmatrix}
\Bigg|
\; \Theta_u \;
\Bigg]
\begin{bmatrix}
\frac{d\lambda(t)}{\lambda(t)} \\
d\alpha_1(t) \\
d\alpha_2(t) \\
\sigma_{Y^0}(t)dw^0(t) \\
\sigma_{Y^1}(t)dw^1(t) \\
\sigma_{Y^2}(t)dw^2(t)
\end{bmatrix}
$$

Same as before

Equilibrium with Portfolio Constraints

Variable/ Effects of	$\dfrac{d\lambda(t)}{\lambda(t)}$	$d\alpha_1(t)$	$d\alpha_2(t)$	$dw^0(t)$	$dw^1(t)$	$dw^2(t)$
$\dfrac{dq^1(t)}{q^1(t)}$	$+$	$+$	$-^{A_1}$	$+$	$-$	0
$\dfrac{dq^2(t)}{q^2(t)}$	$+$	$-^{A_1}$	$+$	$+$	0	$-$
$\dfrac{dS^0(t)}{S^0(t)}$	$-$	$-^{A_2}$	$-^{A_2}$	$+$	$+$	$+$
$\dfrac{dS^1(t)}{S^1(t)}$	$+^{A_3}$	$+^{A_1}$	$-^{A_2}$	$+$	$+$	$+$
$\dfrac{dS^2(t)}{S^2(t)}$	$+^{A_3}$	$-^{A_2}$	$+^{A_1}$	$+$	$+$	$+$

Equilibrium with Portfolio Constraints

- "Transfer Problem":

  – (Keynes vs. Ohlin) If wealth is transferred from one country to another, the terms of trade of the recipient improve.

- Portfolio constraints generate endogenous wealth transfers

  – Transfers are to and from the Periphery

Equilibrium with Portfolio Constraints
Two Examples

- Concentration constraint:

$$x_0^{S^1}(t) + x_0^{S^2}(t) \le \gamma$$

- Market Share Constraint:

$$x_0^{S^1}(t) + x_0^{S^2}(t) \le \gamma \frac{S^1(t) + S^2(t)}{S^0(t) + S^1(t) + S^2(t)}$$

Equilibrium with Portfolio Constraints
Pure Wealth Transfers

- Consider a Concentration constraint. When is the constraint binding?
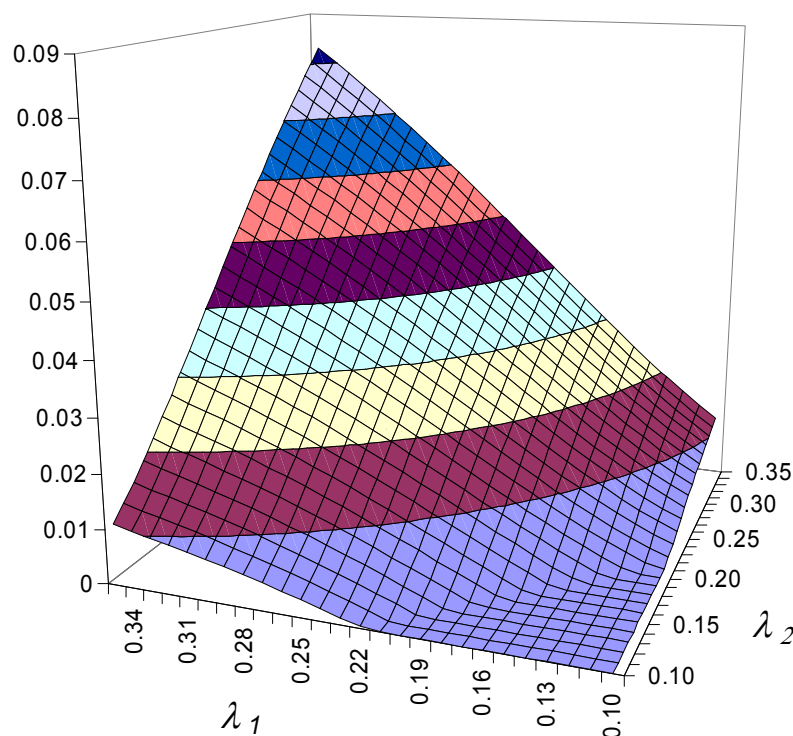
Equilibrium with Portfolio Constraints
Concentration Constraint

- Direction of a wealth transfer for the Transfer Problem is driven by the tilt in the holdings:
  - Center is
    - Over-weighted in the Center stock
    - Under-weighted in the Periphery stocks
  - The opposite for the Periphery countries

- Example: A shock that increases the value of the stock in the Center (leaving the other stocks unchanged) increases the wealth of the Center by more than the wealth of the Periphery countries. Hence λ goes down.

Equilibrium with Portfolio Constraints
Pure Wealth Transfers
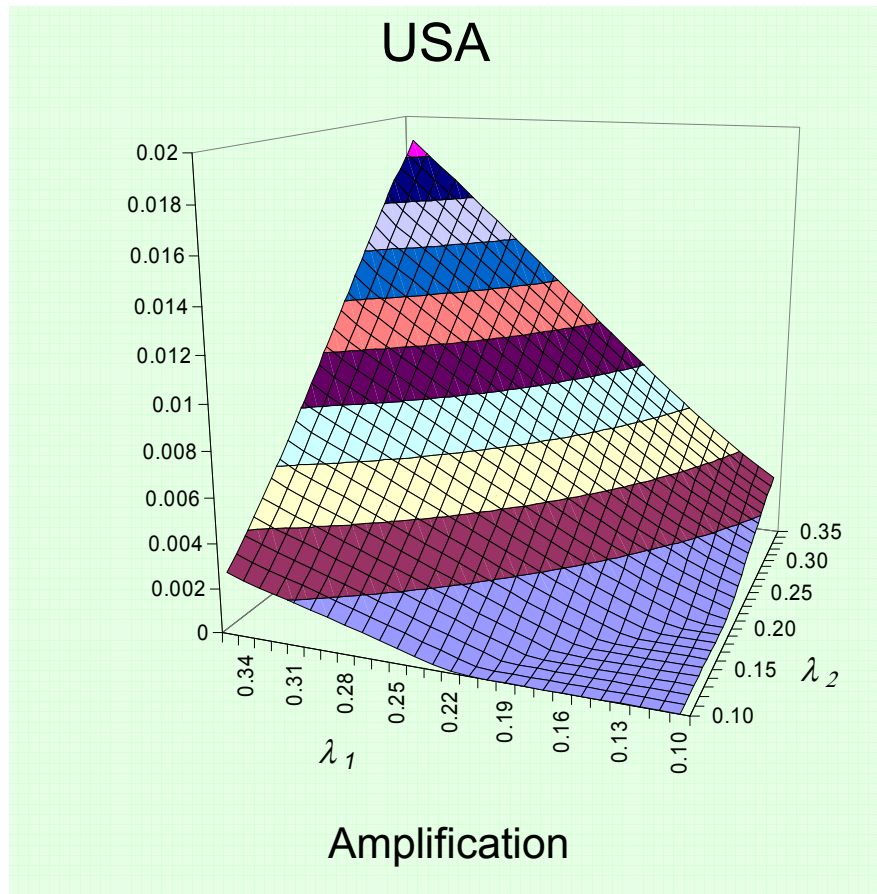
- Impact on λ:
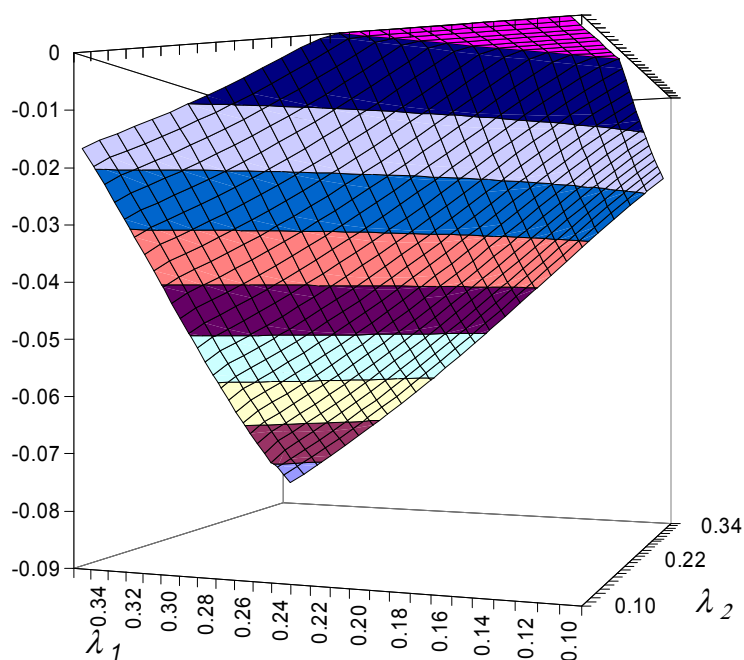
of a shock in the Center
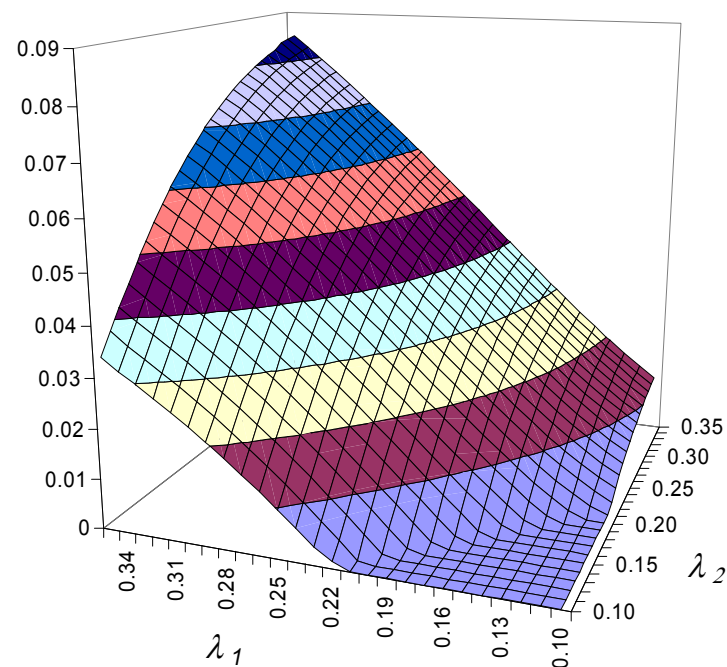
of a shock in the Periphery

Equilibrium with Portfolio Constraints
Pure Wealth Transfers

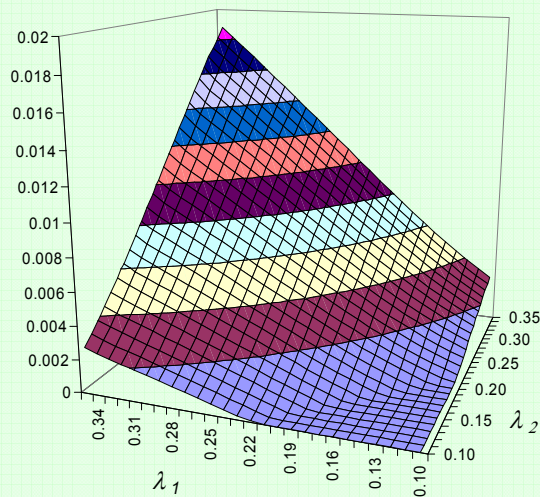- Impact of a shock in the Center on *stock prices* in

USA

Amplification

Periphery
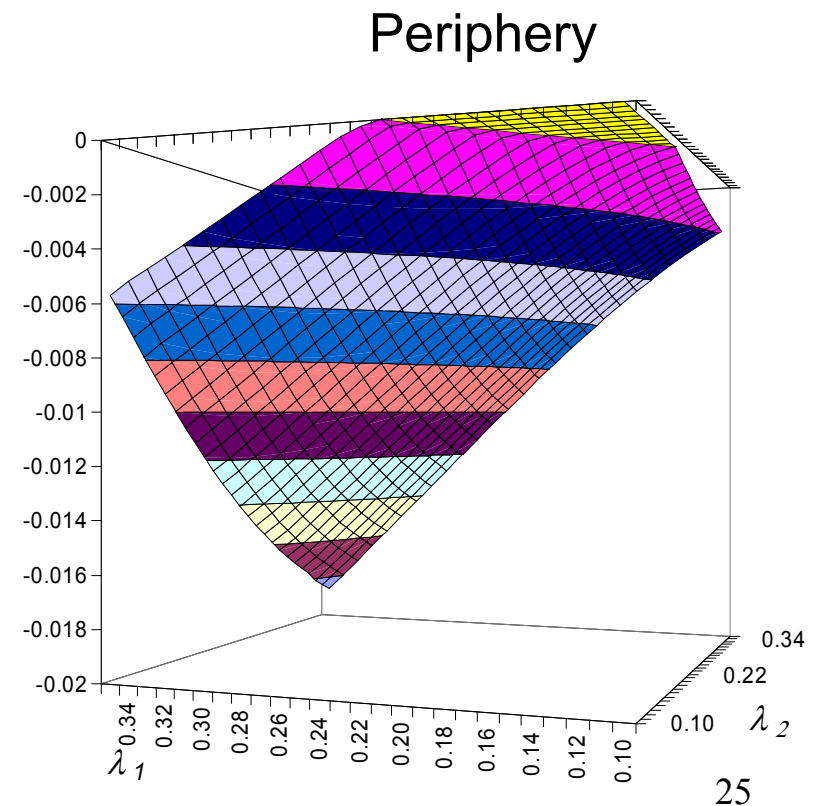
Equilibrium with Portfolio Constraints
Pure Wealth Transfers

- Impact of a *negative* shock in Russia on *stock prices* in

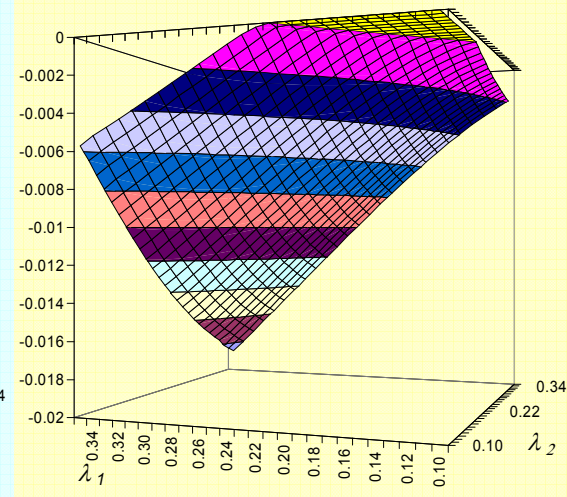 - USA   Russia   Brazil
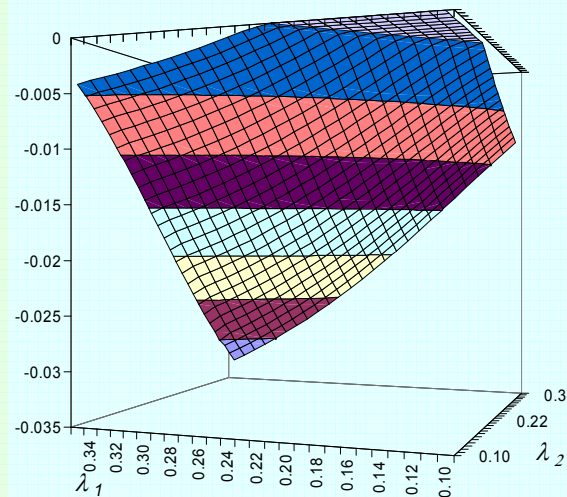
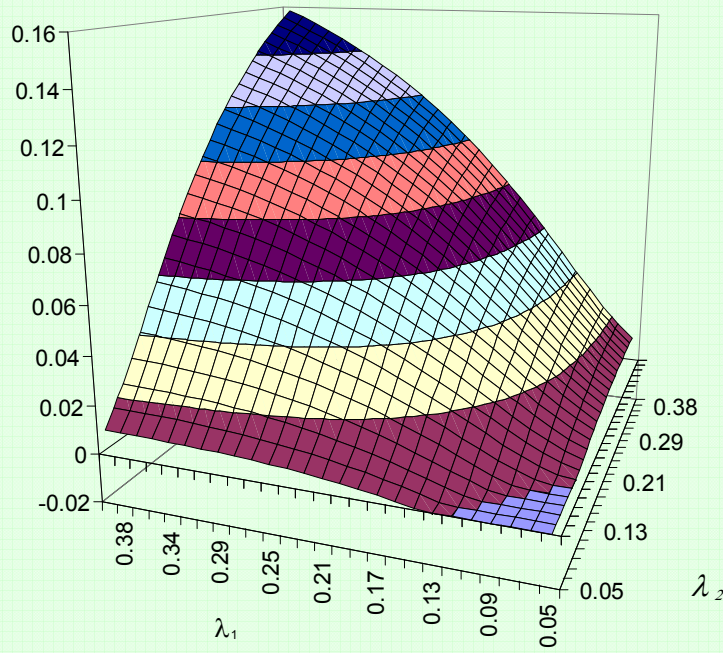Flight to Quality   Amplification   Contagion
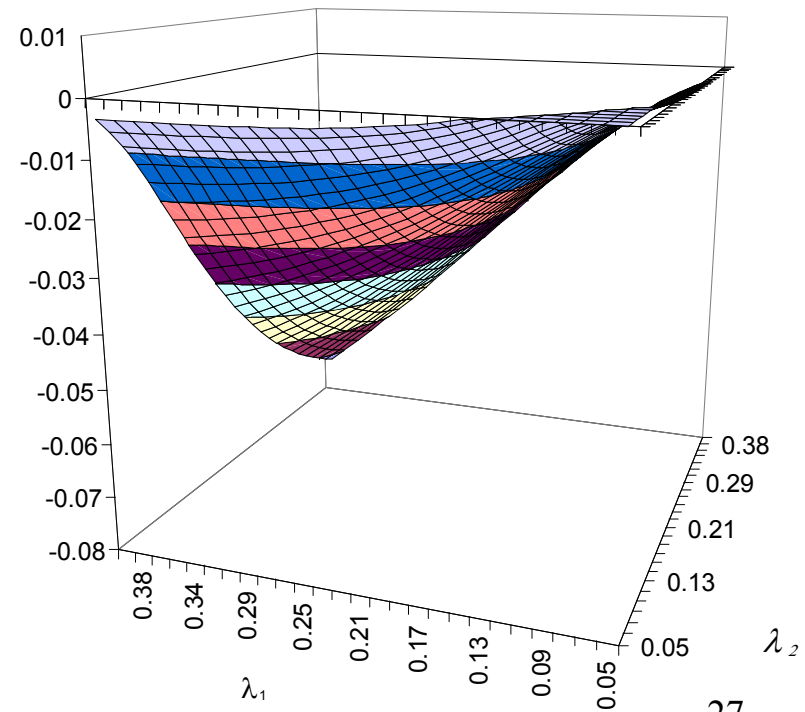
Equilibrium with Portfolio Constraints
Pure Wealth Transfers

- Excess correlations of stock returns


Russia and Brazil

USA and Brazil

Equilibrium with Portfolio Constraints
Pure Wealth Transfers

- Concentration constraint highlights the wealth channel keeping the restrictiveness of the constraint constant.

  – Simple, but might be unrealistic.

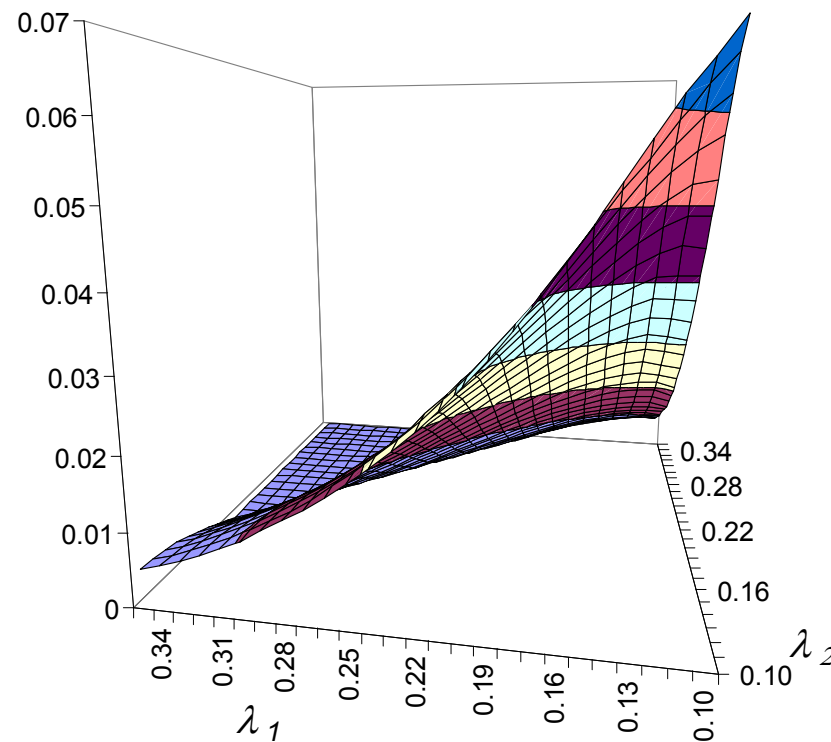 – The constraint loosens in response to a negative shock in the Periphery

Multiplier on the Concentration Constraint, Again

- When is the constraint binding?
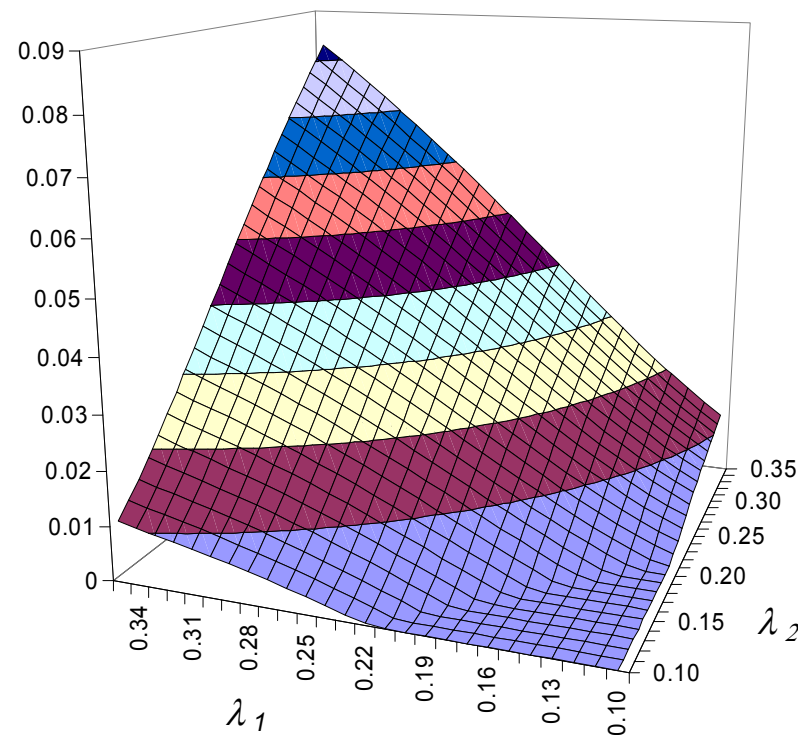
Multiplier on the Market Share Constraint

- When is the constraint binding?

Equilibrium with Portfolio Constraints
Varying Tightness of a Constraint

- A shock produces
  - wealth transfers that depend on the holdings,
    - Same pattern of stock price responses

  - but also changes the tightness of the constraint
    - Patterns of capital flows are more realistic

Conclusion

- We can characterize the dynamics of asset and good prices for any portfolio constraint imposed on the Center.

- The presence of the constraint gives rise to:
  - common factor in stock returns reflecting its tightness
  - wealth transfers among investors
  - the transfers impact terms of trade in the same way they do in the classical "Transfer Problem" of international economics
  - constraints are a source of co-movement of terms of trade and stock prices internationally.

- Concentration and Market Share constraints produce amplification and a flight to quality.

Robustness:
Contagion without Trade

- Assume there is no trade among the Periphery countries

$$u_0\left(C_0^0, C_0^1, C_0^2\right) = \log\left(C_0^0(t)\right)$$

$$u_1\left(C_1^0, C_1^1, C_1^2\right) = \left(1 - \alpha_1(t)\right)\log\left(C_1^0(t)\right) + \alpha_1(t)\log\left(C_1^1(t)\right)$$

$$u_2\left(C_2^0, C_2^1, C_2^2\right) = \left(1 - \alpha_2(t)\right)\log\left(C_2^0(t)\right) + \alpha_2(t)\log\left(C_2^2(t)\right)$$

Robustness:
Contagion without Trade

- Same transmission mechanism:

	$\dfrac{d\lambda(t)}{\lambda(t)}$	$d\alpha_1(t)$	$d\alpha_2(t)$	$dw^0(t)$	$dw^1(t)$	$dw^2(t)$
$\dfrac{dq^1(t)}{q^1(t)}$	+	+	+	+	−	0
$\dfrac{dq^2(t)}{q^2(t)}$	+	+	+	+	0	−
$\dfrac{dS^0(t)}{S^0(t)}$	−	−	−	+	+	+
$\dfrac{dS^1(t)}{S^1(t)}$	+	+	A	+	+	+
$\dfrac{dS^2(t)}{S^2(t)}$	+	A	+	+	+	+