How useful are no-arbitrage restrictions for forecasting the term structure of interest rates?

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- Previous out-of-sample evaluation approaches (Ang and Piazzesi, 2003, Duffee, 2002, Diebold and Li, 2006, Carriero, 2007)
  - Informal comparison of MSFE of no-arbitrage model, unrestricted VAR and random walk
- Our goal: propose a formal framework for investigating the "usefulness" of no-arbitrage restrictions for forecasting

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- Are no-arbitrage restrictions only useful because they reduce estimation error?
- Are the results different when considering economic vs. statistical measures of accuracy?
- How important is the fact that no-arbitrage restrictions incorporate a time-varying risk premium?
- Has the usefulness of no-arbitrage restrictions changed over time?

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  - They compare performance of different *models*
  - We have one model and different restrictions on its parameters

Contributions

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- The measure is tailored to the forecaster's decision problem
- The measure can be time-varying

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- Theoretical restrictions (Ang and Piazzesi's no-arbitrage restrictions)
- Atheoretical restrictions (random walk)
- Statistical and economic loss functions (quadratic and utility of bond portfolio)

The idea in a nutshell

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- Cast the problem in an out-of-sample forecast combination framework
- Combine unrestricted (and random walk) forecast with no-arbitrage restricted forecast and estimate optimal weight for a general loss function
- Optimal weight $=$ measure of the usefulness of no-arbitrage restrictions
- Time-varying weight $=$ time-varying measure of usefulness

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- Two forecast combinations

$$
\begin{aligned}
f_t^c &= f_t^R + (1-\lambda)\left(f_t^U - f_t^R\right) \\
f_t^c &= f_t^R + (1-\lambda)\left(f_t^{RW} - f_t^R\right)
\end{aligned}
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Method. Global measure of usefulness

- For general loss $L(y_{t+1}, f_t)$:

$$\lambda^* = \arg\min_{\lambda \in R} E\left[L\left(y_{t+1}, f_t^R + (1-\lambda)(f_t^U - f_t^R)\right)\right]$$

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- Estimate out-of-sample:

$$\widehat{\lambda} = \arg\min_{\lambda \in R} \frac{1}{n} \sum_t L\left(y_{t+1}, f_t^R + (1-\lambda)(f_t^U - f_t^R)\right).$$

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- Large $\widehat{\lambda}$ = no-arbitrage restrictions are useful
- Small $\widehat{\lambda}$ = no-arbitrage restrictions are not useful

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- Estimate a "smoothed" version of λ_t^* over rolling windows of size d

$$\widehat{\lambda}_{t,d} = \arg\min_{\lambda \in R} \sum_{j=t-d+1}^{t} \left[L(y_{t+1}, f_t^R + (1-\lambda)\left(f_t^U - f_t^R\right)\right]$$

Method. Inference

- Global measure $\Longrightarrow$ test of $H_0 : \lambda = 0$ (restrictions are useless)

$$\text{Reject if } \left| \frac{\sqrt{n}\widehat{\lambda}}{\widehat{\sigma}} \right| > 1.96. \text{ We give valid } \widehat{\sigma}$$

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- Local measure $\Longrightarrow$ uniform confidence bands valid under $H_0 : \lambda_t^*$ constant $\forall t$

$$I = (\widehat{\lambda}_{t,d} - k_{\alpha,\pi}\frac{\widehat{\sigma}}{\sqrt{d}}, \widehat{\lambda}_{t,d} + k_{\alpha,\pi}\frac{\widehat{\sigma}}{\sqrt{d}})$$

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- If $0 \notin I$ at some $t \Longrightarrow$ reject that restrictions were useless $\forall t$

Statistical loss

- For a quadratic loss $L\left(y_{t+1}, f_t\right) = \left(y_{t+1} - f_t\right)^2$ measure of usefulness is

$$\lambda^* = \frac{E\left[\left(y_{t+1} - f_t^U\right)\left(f_t^R - f_t^U\right)\right]}{E\left[\left(f_t^R - f_t^U\right)^2\right]}$$

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- Local measure obtained by estimating rolling regressions
- $\widehat{\sigma}$ for the test and confidence bands is OLS standard error (HAC)

Illustrative example

- Suppose $y_t = \beta' x_t + \varepsilon_t$, $x_t \sim iid(0, \sigma_x^2 I_k)$, $\varepsilon_t \sim iid(0, \sigma_\varepsilon^2)$

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- $\lambda^* \approx 1$ (restrictions useful) if large variance of unrestricted estimator ($\Leftrightarrow$ usefulness due to reduction in estimation uncertainty)

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- Generally true for out-of-sample comparisons based on MSE

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$$w^* = \arg\min_{w} \left\{ w' E_t \left[y_{t+1} \right] - \frac{\gamma}{2} w' \Sigma w \right\}$$

$\Sigma = Var(y_{t+1})$ and $\gamma = CRRA / \left(CRRA + 1 \right)$

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- Classical solution (Markowitz, 1952)

$$\begin{aligned} w^* &= C_1 + C_2 E_t \left[y_{t+1} \right], \\ C_1 &= \frac{\Sigma^{-1} \iota}{\iota' \Sigma^{-1} \iota}, \; C_2 = \frac{1}{\gamma} \left(\Sigma^{-1} - \frac{\Sigma^{-1} \iota \iota' \Sigma^{-1}}{\iota' \Sigma^{-1} \iota} \right) \end{aligned}$$

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- Measure of usefulness

$$\begin{aligned} \lambda^* &= \arg\min_{\lambda \in R} \left[-w^*(\lambda)' E[y_{t+1}] + \frac{\gamma}{2} w^*(\lambda)' \Sigma w^*(\lambda) \right] \\ &= \frac{E\left[(f_t^R - f_t^U)' C_2' \left(y_{t+1} - \gamma \Sigma \left(C_1 + C_2 f_t^U \right) \right) \right]}{E\left[\gamma (f_t^R - f_t^U)' C_2' \Sigma C_2 (f_t^R - f_t^U) \right]} \end{aligned}$$

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- Estimate by substituting $E[\cdot]$ with out-of-sample mean (global) or rolling out-of-sample means (local)

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- F_t contains three latent factors ($\approx$ level, slope and curvature of term structure)

Empirical application. AP model as a restricted VAR

- No-arbitrage assumption restricts the elements of A and B

$$\bar{A}_{n+1} = \bar{A}_n + \bar{B}_n'(-\Lambda_0) + 0.5\bar{B}_n'\Omega\Omega'\bar{B}_n - \delta_0$$
$$\bar{B}_{n+1}' = \bar{B}_n'(\Psi - \Omega\Lambda_1) - \delta_1'$$

with $A_n = -\bar{A}_n/n$, $B_n = -\bar{B}_n/n$, $\bar{A}_1 = -\delta_0$ and $\bar{B}_1 = -\delta_1$

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- Λ_0 and Λ_1 are such that $\Lambda_t = \Lambda_0 + \Lambda_1 F_t$, with $\Lambda_t =$ market prices of risk

$$\begin{aligned}\Lambda_1 \neq 0 &\Longrightarrow \text{time-varying risk premium} \\ \Lambda_1 = 0 &\Longrightarrow \text{constant risk premium}\end{aligned}$$

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- Impose the restrictions on the VAR by writing the sample likelihood of Y as

$$|\Sigma_u|^{-T/2} \exp\{-0.5tr[\Sigma_u^{-1}(\Gamma^*_{yy} - \Phi'\Gamma^*_{xy} - \Gamma^{*\prime}_{xy}\Phi + \Phi'\Gamma^*_{xx}\Phi)]\}$$

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- Estimate by ML $\Longrightarrow$ no-arbitrage restricted estimator
- The unrestricted estimator is the OLS estimator

Empirical results

VERY PRELIMINARY....

Quadratic loss. Global measure of usefulness

NA vs. unrestricted VAR

Yields	$\widehat{\lambda}$	Test of H_0 : NA restrictions useless
1-month	**0.427**	4.711*
3-month	**0.655**	3.547*
12-month	**0.979**	5.374*
36-month	**0.814**	4.823*
60-month	**0.826**	5.424*

Quadratic loss. Global measure of usefulness

NA with constant risk premium vs. unrestricted VAR

Yields	$\widehat{\lambda}$	Test of H_0 : NA restrictions useless
1-month	**0.337**	3.596*
3-month	**0.785**	4.559*
12-month	**0.796**	4.777*
36-month	**0.912**	5.894*
60-month	**0.742**	5.251*

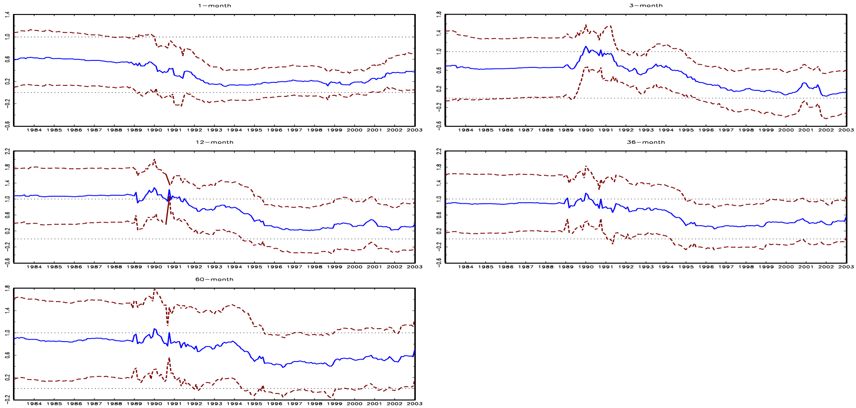
Quadratic loss. Global measure of usefulness

NA vs. random walk

Yields	$\widehat{\lambda}$	Test of H_0 : NA restrictions useless
1-month	**0.516**	4.649*
3-month	**-0.110**	-0.526
12-month	**0.276**	0.828
36-month	**0.099**	0.337
60-month	**0.274**	0.668

Quadratic loss. Local measure of usefulness

NA vs. unrestricted VAR



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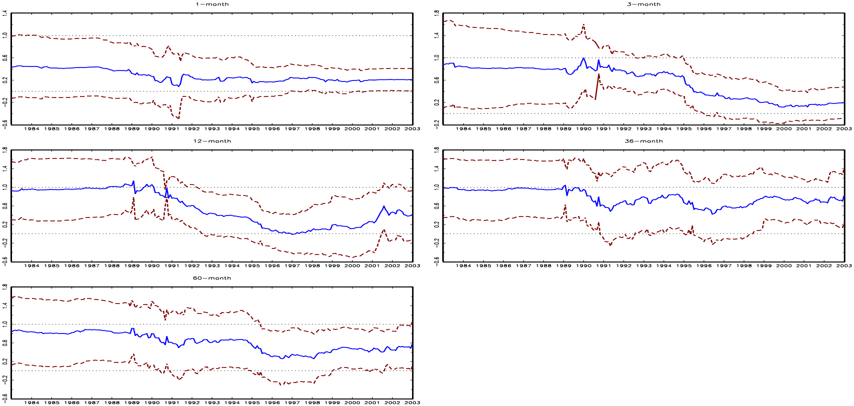

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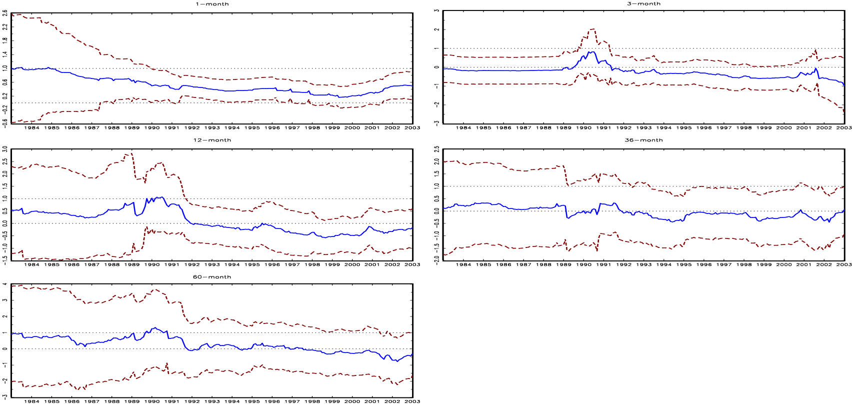


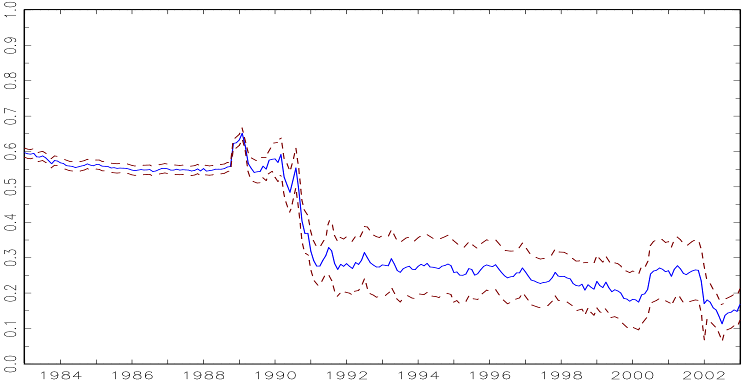
Portfolio utility loss. Global measure of usefulness

Results for CRRA = 1

	$\widehat{\lambda}$	H_0 : NA restr. useless
NA vs. unrestricted VAR	**0.416**	23.33^{*}
NA with const.risk premium vs. unrestricted VAR	**0.199**	26.80^{*}
NA vs. random walk	**0.512**	63.25^{*}

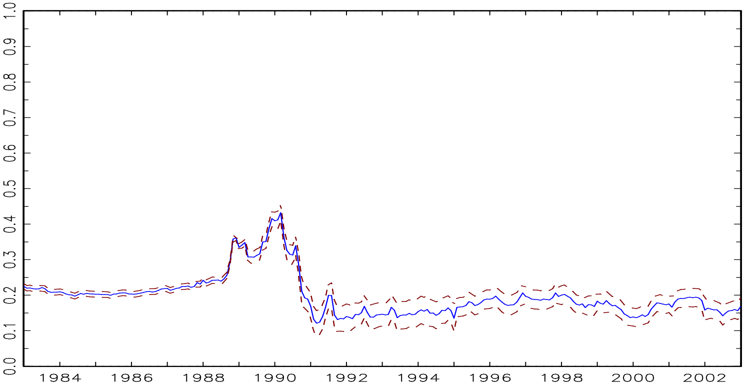
Portfolio utility loss. Local measure of usefulness

NA vs. unrestricted VAR



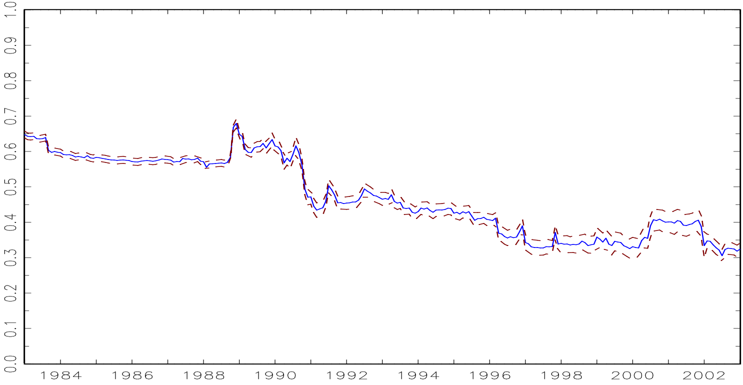
Portfolio utility loss. Local measure of usefulness

NA with constant risk premium vs. unrestricted VAR



Portfolio utility loss. Local measure of usefulness

NA vs. random walk



Conclusion. Theoretical framework

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Conclusion. Back to our initial questions

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  - Not important for statistical accuracy, but essential for constructing bond portfolios
- Has the usefulness of the restrictions changed over time?
  - Yes, they have become less useful