Investment Dynamics with Overconfident Expectations

Ricardo D. Brito

Insper Institute of Education and Research
Associate Professor, Department of Economics
E-mail address: RicardoDOB@insper.edu.br
Phone: 55-11-4504-2431; Fax: 55-11-4504-8415.
Postal address: Rua Quatá, 300, São Paulo, SP 04546-042, Brazil.

Abstract

This paper proposes an investment model in which agents’ overconfidence about the precision of their private information and biased self-attribution (as in Daniel, Hirshleifer and Subrahmanyan, 1998) induce amplified investment cycles. The model is able to generate negative long-lag autocorrelation, excess volatility, and positive short-lag autocorrelation of excess returns. The theory also offers several testable implications.

JEL classification: E22, G1.
Keywords: Capital Investment; Overconfidence; Biased self-attribution, Excess Volatility, Predictability.

\[82\] Preliminary. Please, do not quote or circulate without permission. For valuable discussion, I thank Kent Daniel, Luciana Ferreira and seminar participants at Insper and FACE/UFMG. I also thank Andreas Fuster, Benjamin Hebert, David Laibson, and Brendan Price for kindly providing theirs codes. This project was started while I was visiting Columbia Business School. I thank the Jerome Chazen Institute and Geert Bekaert for their hospitality. All errors are my own. Financial support from the CNPq/Brazil, under grant 313554/2009-9, is gratefully acknowledged.
1. Introduction

Excess volatility of capital investments, as well as excess volatility of financial investment, is widely documented in the literature, in contradiction with the rational expectations-efficient markets benchmark, i.e., real business cycles and non-arbitrage financial pricing theories. Barlevy (2004), for example, shows that a rational model with capital adjustment costs faces difficulty in generating sufficient real investment volatility. In such model, investment growth presents approximately the same volatility as output growth, whereas investment growth in the data is about two times more volatile than output growth. Regarding financial assets, Shiller’s (1981) evidence that stock prices move too much to be justified by changes in dividends is widely known.

Besides excess volatility, negative long-lag autocorrelation (overreaction) and positive short-lag autocorrelation (momentum) of investment and returns also challenge the neoclassical theory of investments, which additionally does not explain the booms and busts occurring regularly in economic activity.¹

The objective of this article is to propose a theory of overconfident capital investment that is able to endogenously generate high physical investment volatility, negative long-lag autocorrelation, and positive short-lag autocorrelation.

How do optimism/pessimism and overconfidence affect risk taking, investment and growth?

Basically, individuals that have recently observed high (low) investment returns present extrapolation bias and are more willing to take new bets in the same direction. The more successful the over-optimist (over-pessimist) investors are, the more confident they become of their valuation skills, and the faster and riskier they invest, similar to Keynes’ (1936) “animal spirits”.

Is there support for such behavior?

The premise of investor overconfidence is derived from a large body of evidence, which shows that individuals overestimate their own abilities in various contexts (see for example,

¹ Real business cycles and efficient financial market theories do not endogenously explain the booms and busts phenomena, usually invoking to exogenous shocks.
DeBondt and Thaler (1995) and Odean (1998)). If an investor overestimates his ability to generate information, he will underestimate his forecast errors.

Like Daniel, Hirshleifer and Subrahmanyam (1998), I assume that when an investor receives confirming public information, his confidence rises, but disconfirming information causes confidence to fall only modestly, if at all. Thus, if an individual begins with unbiased beliefs about his ability, new public signals on average are viewed as confirming the validity of his private signal.

The premise of self-attribution bias is also based in psychology that reports that as individuals observe the outcomes of their actions, they update their confidence in their own ability in a biased manner. Individuals attribute events that confirm the validity of their actions to high ability, and events that disconfirm the action to external noise (see for example, Langer and Roth (1975) and De Long et al. (1991)). Like Daniel, Hirshleifer and Subrahmanyam (1998), I assume that when an investor receives confirming information, his confidence rises, but disconfirming information causes confidence to fall only slightly.

The proposed investment behavior accords with: (i) Giuliano and Spilimbergo (2009), who show that individuals growing up during recessions tend to believe that success in life depends more on luck than on effort; (ii) Malmendier and Nagel (2011), that find that individuals who have experienced low stock market returns are less willing to take financial risk and are more pessimistic about future stock returns; and Malmendier and Tate (2005), which report that overconfident managers overestimate the returns to their investment projects.

In a q-model of investment, with linear profit function and quadratic adjustment costs, I assume that agents are overconfident and biased in their evaluation of the total factor productivity. Given the difficulties agents have to distinguish between permanent and transitory productivity shocks, documented in Croce (2010), Edge et al. (2007), and Beaudry and Portier (2006) for example, it seems a sensible hypothesis to start with.

By doing so, I endogenously generate excess volatility in the real investments, a desirable feature, given Blanchard Rhee and Summers (1993) find a limited role for financial market valuation causation in managers’ capital investment decisions. Such an overconfident investment behavior also has potential use in general equilibrium production-based asset pricing models, like Cochrane (1991) and Jermann (1998), to help to match stylized facts about both macroeconomic aggregates and asset prices.
With overconfidence and self-attribution bias, the volatility of investment increases, and it is possible to explain both optimism and pessimism, without appealing to disasters or time varying aggregate uncertainty.² It proposed model is also an alternative to the accelerator mechanism of Bernanke, Gertler and Gilchrist (1999).

A similar article to this one is Fuster, Herbert and Laibson (2012), who also use a partial equilibrium q-model of investment, but with agents that have wrong beliefs about the rate of mean reversion. For comparison, the results for their model are also shown below.

In addition to this introduction, I develop the model in the following section, present the simulation results in section 3 and conclude in section 4, with some thoughts for future research.

2. The Model

Consider a risk-neutral competitive firm \( i \) at time \( t \) that has capital level \( k(i,t) \), faces quadratic investment costs \( C(I(i,t)) \) and earns constant returns to scale revenues \( \pi(K(t), TFP(t))k(i,t) \), with:

\[
\pi(K(t), TFP(t)) = 1 - K(t) + TFP(t), \tag{1}
\]

and

\[
C(I(i,t)) = \frac{\alpha}{2} I(i,t)^2, \tag{2}
\]

² For example, Cogley and Sargent (2008) only model pessimism. Barro (2006), Rancière et al. (2008) and Jones (2011) model disasters, and Bloom (2009) models time varying aggregate uncertainty. However, in these set ups, active risk exposure decreases when countries get richer, which is at odds with the origination of the 2007-2008 crisis!
where $K(t)$ is the aggregate capital and $\overline{TFP}(t) = (\bar{\eta} + \bar{\eta}_t)$ is an exogenous random total factor productivity measure, both taken as exogenous by firms.

For a fixed set of identical firms on the unit interval, $i \in [0,1]$, the following equalities hold for aggregate capital:

$$K(t) = \int_0^1 k(i, t) di = k(t),$$  \hspace{1cm} (3)

and investment:

$$\dot{K}(t) = \int_0^1 I(i, t) di = I(t).$$  \hspace{1cm} (4)

Such firm acts to maximize its net present value:

$$NPV(t) = E_t \int_{s=t}^{\infty} \left[ \pi(K(s), TFP(s))k(s) - I(s) - C(I(s)) \right] \exp(-r(s - t)) ds,$$  \hspace{1cm} (5)

subject to:

$$\frac{dk(t)}{dt} = I(t).$$  \hspace{1cm} (6)

Its value function, defined as the maximized net present value, is:
\[ V(k(t), K(t), TFP(t)) = \max_{I(\tau)} E_t \left[ \int_{\tau=t}^{\infty} \left[ \pi(K(s), TFP(s))k(s) - I(s) - C(I(s)) \right] \exp(-r(s - t)) ds \right], \quad (7) \]

or, its continuous-time Bellman equation version:

\[ rV(k, K, F) = \max_{\tau} \left[ \pi(K, TFP)k - I - C(I) + E_t \left[ \frac{dV}{dt} \right] \right]. \quad (8) \]

The first order conditions are:

\[ 1 + C'(I(t)) = q(t), \quad (9) \]

and

\[ rq(t) = \pi(K(t), TFP(t)) + E_t \left[ \frac{dq}{dt} \right], \quad (10) \]

where:

\[ q(s) = \frac{\partial V(k(s), K(s), F_P(s))}{\partial k(s)} = E_t \left[ \int_{\tau=t}^{\infty} \pi(K(s), TFP(s)) \exp(-r(s - t)) ds \right] \quad (11) \]

is the marginal present value of a unit of installed capital.
Equation (9) requires that the marginal cost of acquiring and installing capital equals the marginal present value of capital. Equation (10) requires that the interest forgone on the value invested in the marginal unit of capital equals the flow of revenues plus the expected capital gain.

So far, I have presented a standard q-model of investment with uncertainty. What differentiates the proposed model is that overconfident and self-attributed investors will calculate their expectation of next period total factor productivity similar to Daniel, Hirshleifer and Subrahmanyam (1998).

Daniel, Hirshleifer and Subrahmanyam (1998) propose a theory of securities market under- and overreactions based on two well-known psychological biases: (i) investor overconfidence about the precision of private information; and (ii) biased self-attribution, which causes asymmetric shifts in investors' confidence as a function of their investment outcomes.

In a dynamic endowment economy, they show that: (i) overconfidence implies (i.i) negative long-lag autocorrelations (overreaction), and (i.ii) excess volatility; and (ii) biased self-attribution adds (ii.i) more volatility and (ii.ii) positive short-lag autocorrelations (“momentum”), but (ii.iii) negative correlation between future returns and long-term past stock market performance.

I adapt Daniel, Hirshleifer and Subrahmanyam (1998) by calculating the expectation of the total factor productivity with overconfidence and self-attribution as:

\[
E_{c,t}[TFP(t + \tau)] = \frac{(t - 1)\nu_\eta \Phi_t + v_{c,t} s_1}{v_\theta + (t - 1)\nu_\eta + v_{c,t}},
\]

where:

\[
\Phi_t = \frac{1}{(t - 1)} \sum_{\tau = 2}^{t} \bar{TFP}_\tau = \bar{\theta} + \frac{1}{(t - 1)} \sum_{\tau = 2}^{t} \bar{\eta}_\tau,
\]
\[
\begin{cases}
\text{sign}(s_1 - \Phi_{t-1}) = \text{sign}(\phi_t - \Phi_{t-1}) & \text{and} \quad |s_1 - \Phi_{t-1}| < 2\sigma_{\Phi_t} \\
\text{otherwise} & \quad \text{then} \quad v_{c,t} = (1 + k)v_{c,t-1}
\end{cases}
\] (14)

and without overconfidence or self-attribution as:

\[
E_t[\text{TFP}(t + \tau)] = \frac{(t - 1)v_\eta \Phi_t + v_\epsilon s_1}{v_\theta + (t - 1)v_\eta + v_\epsilon}.
\] (15)

Intuitively, equations (12) to (15) capture the agents’ inability to distinguish in real time between transitory and permanent shocks to the TFP. For example, given an initial positive shock, biased and unbiased agents update their beliefs about its permanent nature with the observation of the subsequent realizations of the TFP, respectively represented by equations (12) and 15). If the values of the following TFPs are still high, than the initial shock is more likely to be a permanent improve. However, along this updating process, the self-attribution biased investors that had a positive prior about the TFP, become overoptimistic about it as receives confirming information.

Although stochastic, the solution of the above system does not pose much difficulty. As shown in Abel (1985), under this case where the \( \pi(K, \text{TFP}) \) is linear, the uncertainty concerns the intercept of the function and adjustment costs are quadratic, the market value of capital is the same with the uncertainty as it is if the future values of the \( \pi(K, \text{FP}) \) function are certain to equal their expected values. Given invest is driven by the market value of capital, investment is also the same with the uncertainty as it is if the future values of the \( \pi(K, \text{FP}) \) function are certain to equal their expected values.

Given the vector:

\[
y(t) \equiv \begin{pmatrix} q(t) \\ K(t) \\ \text{TFP}(t) \end{pmatrix},
\]
the evolution of the system can be expressed as:

\[
\frac{dy(t)}{dt} = A + By(t) + \varepsilon(t) = \left( \begin{array}{c} -1 \\ \frac{1}{\phi \theta} \\ \frac{r}{1} \\ 0 \\ 0 \\ -\phi \end{array} \right) y(t) + \left( \begin{array}{c} 0 \\ \eta_t \\ 0 \\ 0 \\ 0 \\ \bar{\eta}_t \end{array} \right).
\] (16)

Basically, the model is saying that the realized total factor productivity \( TFP(t) = (\theta_t + \eta_t) \), has a mean which agents have difficulties to get right, given \( \theta_t \) is always mixed with the noise \( \eta_t \). Every variation they observe in the total factor productivity, they wonder whether it is just noise, or a change in the mean of \( \tilde{\theta} \).\(^3\) Given they recently observed a positive permanent change in the total factor productivity, every time they observe a positive shock, they get more confident that they are experiencing another positive permanent change in the total factor productivity. But given, this time the shocks are actually just noise, their over-optimism eventually goes away.

Fuster, Herbert and Laibson (2012) model the agents’ mistake differently. Their agents know from the very beginning that the shock is not permanent but temporary. However, they underestimate the speed of mean reversion, assuming \( \phi \).

Define the vector \( y(\infty) \), and calculate:

\[
E_{c,t}[y(\infty)] = \left( 1 - \frac{r + E_{c,t}[TFP(\infty)]}{E_{c,t}[TFP(\infty)]} \right),
\] (17)

and:

\(^3\) I have just come across Grenadier and Malenko’s (JF 2010) paper, “A Bayesian Approach to Real Options: the case of distinguishing between temporary and permanent shocks”. They use Bayesian updates when agents cannot distinguish between transitory or permanent shocks, however without any bias. Their model is closer to what I call “Balanced”.
\[ E_t[y(\infty)] \equiv \left( \frac{1}{1 - r + E_t[TFP(\infty)]} \right), \]  

(18)

with the steady state condition:

\[ BE_{c,\infty}[y(\infty)] = BE_\infty[y(\infty)] = -A. \]  

(19)

Assuming convergence, the rational expectation of \( y(t + \tau) \) can be expressed as:

\[ E_t[y(t + \tau)] = E_t[y(\infty)] + \exp(B\tau)M(y(t) - E_t[y(\infty)]), \]  

(20)

and the overconfident expectation as:

\[ E_{c,t}[y(t + \tau)] = E_{c,t}[y(\infty)] + \exp(B\tau)M(y(t) - E_{c,t}[y(\infty)]), \]  

(21)

where \( M = VL'(LVL')^{-1}L \), with matrix \( M = VL'(LVL')^{-1}L \) containing the eigenvectors of \( B \) in decreasing order and:

\[ L = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \]

Note that:

\[ y(t) = E_t[y(\infty)] + M(y(t) - E_t[y(\infty)]), \]
\[ E_t[y(t + \tau)] = E_t[y(\infty)] + \exp(MB\tau)M(y(t) - E_t[y(\infty)]). \]

3. Simulation results

3.1. Calibration

In the calibration below, we use Daniel, Hirshleifer and Subrahmanyam (1998) parameters to model overconfident and self-attribution bias. I assume \( \bar{k} = 0.75 \), \( k = 0.05 \), \( \sigma_{\theta}^2 = \sigma_{\epsilon}^2 = 1 \), \( \sigma_{\eta}^2 = 7.5 \).

For comparison purposes, I follow Fuster, Hebert and Laibson (2012) parameter whenever possible. Thus, the risk free rate is \( r = 0.05 \), the adjustment cost parameter is \( \alpha = 10/(1 - r) \), the initial total factor productivity is \( TFP(1) = 0 \), and the true rate of mean reversion is given by \( \phi = 0.25 \). With these parameters, \( K_{\infty} = 0.95 \).

Additionally, in Fuster, Hebert and Laibson (2012) simulation case, named “Natural Expectations”, agents underestimate it to be \( \phi = 0.05 \).

In the simulations below, I assume the initial shock value is equal to \( 0.2375 \), what can be interpreted as the product of higher long-term total factor productivity \( E[TFP(\infty)] = 0.095 \) by the mean reversion speed \( \phi = 0.25 \), or just a transitory shock. A \( TFP(\infty) = 0.095 \) would cause a 10% increase in the capital stock if it were a permanent shock. Like in Daniel, Hirshleifer and Subrahmanyam (1998), this initial shock is just noise and agents will realize that by observing other realizations of \( TFP(t) \).

3.2. Impulse response functions

The figures below report the impulse response functions of the variables involved in the investment decision.
In Figure 1, similar to Daniel, Hirshleifer and Subrahmanyam (1998), I first illustrate the average expected total factor productivity path following a positive private information signal \( \text{TFP}(1) = (\theta + \eta_1) = 1 \) that makes agents optimists. Assuming the private signal is just noise and there is no change in the mean \( \theta = 0 \) of the total factor productivity \( \text{TFP}(t) \), the agents will gradually realize that \( \bar{\theta} \) is still 0, as they observe other public realizations of \( \bar{\text{TFP}}(t) \).

The difference between overoptimistic behavior and the rational benchmark is that overoptimistic agents with self-attribution bias get more confident that there has been a positive change in the mean total factor productivity, as they observe positive \( \text{TFP}(t) \)s and almost ignore the non-positive ones. The overoptimistic investors eventually realize they are wrong and also converge to the right fundamental.

Figure 2 presents the evolution of the total factor productivity expected by various kinds of agents when the observed total factor productivity increases by \( d\text{TFP}(1) = 0.25 \times 0.95 \). This may mean: (i) that \( \bar{\theta} \) permanently increased by 0.95, (ii) that there was a transitory shock \( \eta_1 = 0.25 \times 0.95 \), or (iii) a mixture of both. Named “Permanent” is the trajectory that productivity would follow had the shock been permanent and identified by the agents as such since \( t = 1 \). Named “Transitory” is the trajectory that productivity would follow had the shock been transitory and identified by the agents as such since \( t = 1 \). All the “Forecasts” are trajectories that productivity would follow if the studied agents (that have wrong beliefs) were right about their expectations. The “Balanced Forecast” follows the update rule described in equation (15), which attributes sensible weight to the possibility of the TFP shock being permanent. There is no bias in this case, but just the fact that agents are not certain about the data generating process. The “Overconfident Forecast” follows the update rule described in equations (12)-(14) and present self-attribution bias. Finally, the “Natural Expectations Forecast” is taken from Fuster, Hebert and Laibson (2012) and underestimate the speed of mean reversion of productivity shocks, mistakenly taking \( \phi = 0.05 \), instead of \( \phi = 0.25 \).

It is important to note that total factor productivity is an exogenous process not affected by agents’ expectations, and thus it will always follow its true dynamics, represented by the “Transitory” positive shock in in the simulations below. This is without loss of generality. Permanent and negative shocks could be simulated as well and would provide similar intuition.
Figures 3.a and 3.b show the evolution of the market value of installed capital, $q(t)$. If agents knew from the very beginning that the technology shock was temporary and would mean-revert to zero, $q(t)$ would follow the “Transitory” trajectory (dotted black). Basically $q(t)$ would jump up because there are temporary profits to be made with more capital. And then capital would fall, going below one, before returning to the unit steady-state value. This overshoot on the way down is due to the excess of capital that has to be decumulated as productivity falls, and is present in all trajectories, rational or not. Because of uncertainty about the permanent or temporary nature of the shock, $q(t)$ goes up more in the “Balanced” case than in the “Transitory” one, taking longer to return to unit. And because of added overconfidence, $q(t)$ goes up even more and takes even longer to revert to unit in the “Overconfident”. Given the initial uncertainty about the fundamental change in the TFP and overconfidence caused overinvestment, $q(t)$ also falls below unit once the investors realize the shock was transitory. Comparing the “Overconfident” dynamics with the “Natural Expectations” one (i.e., Fuster, Hebert and Laibson (2012) case), it is noticeable that the market value of capital fluctuates more and takes longer to converge in the former.

In Figures 4.a and 4.b, the one period excess returns, defined as $\left(\pi(K(t), TFP(t)) + E_t[\dot{q}(t)]\right)/q(t) - r$, are presented, omitting the “infinite” positive rate of return with the initial shock. There are no excess returns for the “Transitory” path, given agents have the right beliefs about the true process. However, for the “Balanced”, “Overconfident” and “Natural” trajectories, there are long series of negative excess returns. I HAVE TO THINK BETTER ABOUT THE INTUITION FOR THESE DIFFERENT DYNAMICS.

The period flow profits, defined as $\left[\pi(K(t), TFP(t))k(t) - I(t) - C(I(t))\right]$, are plotted in Figure 5. As expected, profits jump up with the initial positive shock and then drift back down because of subsequent capital accumulation and TFP reversion to zero. In the transitory case, the convergence to steady-state is almost monotonic, with a subtle overshooting, what makes profits positively autocorrelated. In the “Balanced”, “Overconfident” and “Natural” trajectories, there are more overshooting, what generates long-term negative autocorrelation. Again due to greater excess of capital, overshooting is larger in the “Balanced” case and even more in the “Overconfident” one.
Figure 6 presents the evolution of the aggregate capital. Given the TFP shock is actually temporary, capital follows a hump-shaped pattern. However, wrong beliefs cause the hump to be more pronounced and the convergence to the steady-state to be slower. One more time, the “Overconfident” case is the one with larger amplitude and phase, followed by the “Balanced” case, and then by the “Natural” case.

Finally, Figures 7.a, 7.b and 7.c show the dynamics in the $K$-$q$ space. The starting point is the lower left corner, where $q(0) = 1$ and $K(0) = 0.95$. With the initial shock, $q(1)$ jumps up, the greater the jump the more agents expect the TFP to persist high. Were the shock “Permanent”, capital would have a higher steady-state. However, the illustrated shock is “Transitory” and, although capital is going to grow to capture the current higher TFP, it should be temporary, eventually returning to the initial steady-state. This capital fluctuation, away and back to the initial steady-state also happens for the wrong beliefs causes, however with larger amplitude. And, as explained in Figures 3 and 6, with the largest variation for “Overconfident” expectations, followed by “Balanced” expectations, and then by “Natural” ones.

In sum, it seems that agents’ real time uncertainty about the permanent-temporary nature of the TFP shock – not necessarily a bias – is able to generate the same qualitative pattern and more volatility than observed in Fuster, Hebert and Laibson (2012), who instead assume wrong beliefs about the speed of mean reversion. If one adds overconfidence to the real time uncertainty about the permanent-temporary nature of the TFP shock, then it is possible to generate even more volatility.

4. Conclusion

This article shows that an investment model in which agents’ overconfidence about the precision of their private information and biased self-attribution induce amplified investment cycles.
The model is able to generate negative long-lag autocorrelation, excess volatility, and positive short-lag autocorrelation of excess returns. Corporate earnings present short-run drift, but there is negative correlation between future returns and long-term past performance.

Although these results are qualitatively similar to Fuster, Hebert and Laibson (2012), they are quantitatively larger, thus making it easier generate the stylized facts of investment departures from the rational expectations benchmark. Actually, this article has shown that to generate such departures, it is not necessary to resort to some kind of irrationality. The very uncertainty of the permanent-transitory nature of the shock is sufficient to accommodate those. The inclusion of overconfidence, however, considerably amplified the fluctuations, a desirable property.

Future versions of this paper will redo the computations in discrete time on a firm-by-firm basis, instead of in continuous time for the aggregate of firms after computing the average expected value of productivity (as it has been done in this version). The updating process of the expected TFP also has to be refined to correctly incorporate a mean-reverting process. And, besides the comparison with Fuster, Hebert and Laibson (2012), comparisons with Christiano’s et al. (2008) and Hirshleifer and Yu’s (2012) proposed expected TFP dynamics should be added. Finally, real data observed dynamics should be the metric to decide among the competing models.

In future research, my intention is to incorporate such mechanism into a general equilibrium consumption-production model, like Cochrane (1991) or Jermann (1998). Similar attempts are already being made, like Christiano et al. (2008) or Hirshleifer and Yu (2012), but have appealed to additional sources of volatility (see Kaltenbrunner and Lochstoer (2010), on which Hirshleifer and Yu (2012) is based).

The interaction of overconfidence and self-attribution bias has shown to be able to generate considerable volatility in investment and production and seems a promising road to follow.
References:


Figures:

Figure 1. Average price path following private information shock

Figure 2. Impulse response function of productivity
Figure 3.a. Impulse response functions for $q$

![Impulse response functions for q](image1)

Figure 3.b. Zoom in of the impulse response functions for $q$

![Zoom in of impulse response functions for q](image2)
Figure 4.a. Impulse response functions for excess returns

Figure 4.b. Zoom in of the impulse response functions for excess returns
Figure 5. Impulse response functions for flow profits

Figure 6. Impulse response functions for capital
Figure 7.a. K-q diagram for impulse response

Figure 7.b. Zoom in K-q diagram for impulse response
Figure 7.c. K-q diagram for impulse response with forecasts